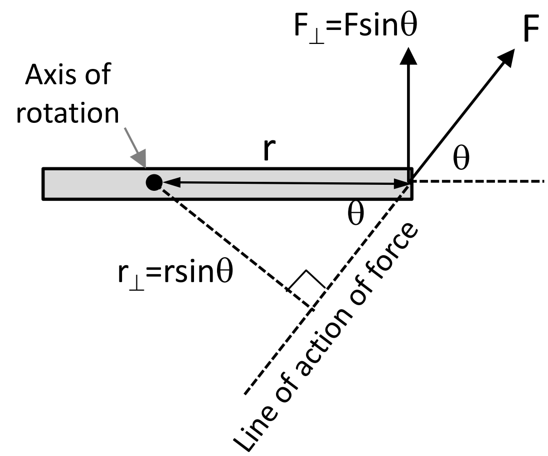
TORQUE - Translational and Rotational Equilibrium Problems

Forces change the velocity of an object. But what does it take to change the angular velocity of an object? Forces are involved, but the force has to be applied in a special way. We call this special applied force a ***torque***.

Torques change angular velocity. The symbol for torque is the Greek letter τ. The torque produced by a force, F, is given by the following equation:

$$\tau = rF\sin\theta = rF_{\perp} = r_{\perp}F$$



r is the distance between the rotation axis and the point of application of the force

$r_{\perp}$ is often called the ***lever arm***.

$F_{\perp}$ is the force component that is perpendicular to r. It is the component of the force that produces a torque. The parallel component of the force produces no torque.

Equilibrium

An object is in ***translational equilibrium*** if there is no net force acting on it: $\sum \vec{F} = 0$

This is a vector equation so it has to be applied in each direction separately.
An object in translational equilibrium is at rest or moving at a constant linear velocity.

An object is in ***rotational equilibrium*** if there is no net torque acting on it: $\sum \vec{\tau} = 0$

An object in rotational equilibrium is at rest or rotating at a constant angular velocity

Important in solving problems: If an object is in rotational ***STATIC equilibrium*** (at rest), the net torque about ANY rotation axis is zero. Therefore, you can choose your axis to be ANYWHERE - choose a point that is most convenient for solving the problem (a point where you don't know much about the forces).

There are ***three common types of equilibrium (Rotational and Translational) Problems***

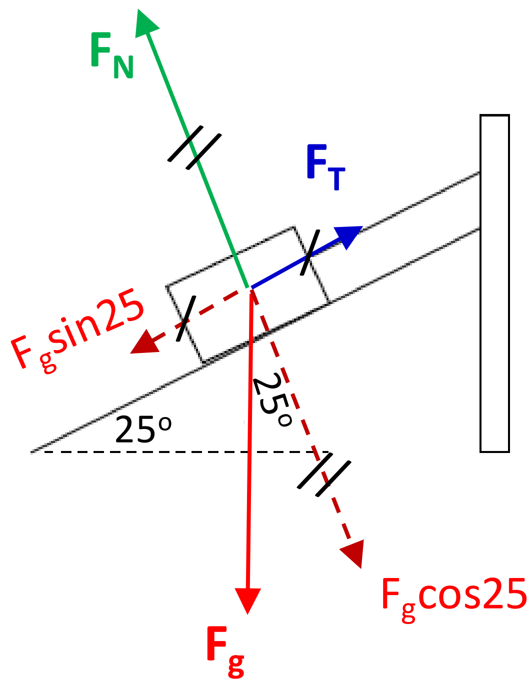
1. Plank or SeeSaw problems
2. Shelf or crane problems (beam sticking out of wall)
3. Ladder problems

Translational Equilibrium Problems (Review from chapter 4)

Use separate sheets of paper to solve these problems. Practice showing the steps of your work clearly, starting from a fundamental equation (Newtons 2nd law). For each problem draw a free body diagram before solving.

1. An object is suspended on a frictionless inclined plane by a rope parallel to the incline as shown. The angle of the incline is 25° and the tension in the rope is 5000 N,
   a) Draw the free body diagram
   b) Determine the weight of the object.

   (***VERY Important Note*** – We can treat the object as a point particle rather than an extended object since all the forces acting on it act at the same point (its center of gravity). Since the forces go through the same point (CG), each force produces NO torque on the object and this is just a translational problem. You do not need to consider torque at all.)



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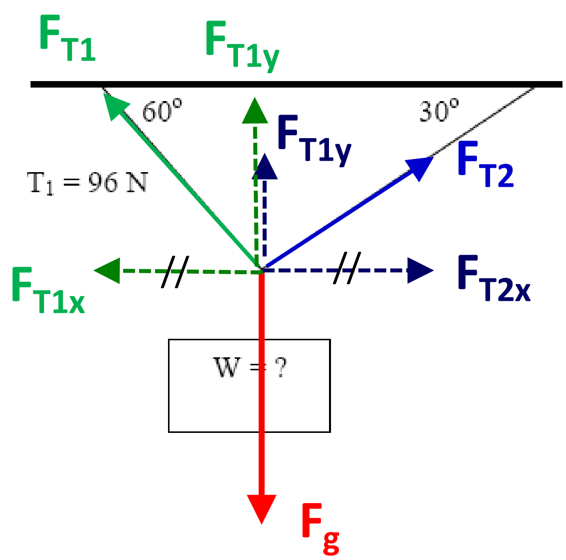
Block is in equilibrium, at rest, so there is no net force on it.

$$\sum F_x = 0$$
$$-F_g \sin 25 + F_T = 0$$
$$F_g \sin 25 = F_T = 5000$$
$$F_g = 11{,}830 N$$

2. Given the diagram at right,
   a) Draw the free body diagram
   b) Find W and T2.

   (***VERY Important Note*** – We can treat this system as a point particle rather than an extended object. Since all the tension forces act at a single point and the system is in equilibrium, that intersection point is the center of gravity of the system. Since all the forces go through the same point (CG), each force produces NO torque and this is just a translational problem. You do not need to consider torque at all.)



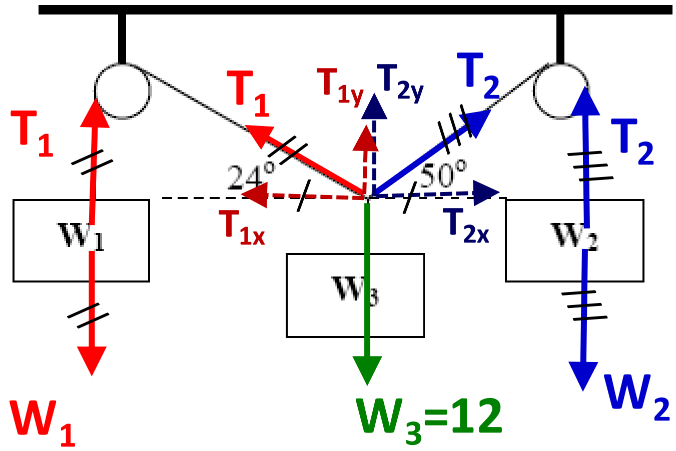
Block is in equilibrium, at rest, so there is no net force on it

$$\sum F_x = 0$$
$$F_{T1x} = F_{T2x}$$
$$F_{T1} \cos 60 = F_{T2} \cos 30$$
$$96 \cos 60 = 0.866 F_{T2}$$
$$F_{T2} = 55.4 N$$

$$\sum F_y = 0$$
$$F_{T1y} + F_{T2y} - F_g = 0$$
$$F_{T1} \sin 60 + F_{T2} \sin 30 = F_g$$
$$111 N = F_g$$

3. W1, W2 and W3 are the weights of three objects suspended by pulleys as shown. W3 = 12 N and assume that the pulleys in this system are frictionless and weightless
   a) Draw the free body diagram
   b) Determine the values of W1 and W2

(***<u>VERY Important Note</u>*** – since the pulleys are weightless and frictionless, they just bend the direction of the tension force (which is equal to the weight it supports) without altering its magnitude. Therefore, all the forces can be drawn as acting on the intersection of the ropes. Since <u>all the forces go through the same point</u> and the system is in equilibrium, each force produces NO torque and this is a translational problem. You do not need to consider torque at all. You can treat the whole system as a point particle rather than an extended object.)



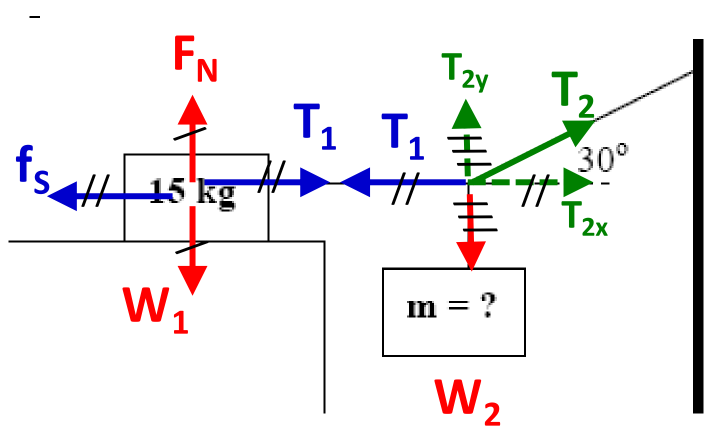
the

$$\sum F_x = 0$$
$$T_{1x} - T_{2x} = 0$$
$$T_1 \cos 24 = T_2 \cos 50$$
$$T_1 = 0.7036 T_2$$

$$\sum F_y = 0$$
$$T_{1y} + T_{2y} - W3 = 0$$
$$T_1 \sin 24 + T_2 \sin 50 - 12 = 0$$
$$(0.7036 T_2) \sin 24 + T_2 \sin 50 = 12$$
$$0.286 T_2 + 0.766 T_2 = 12$$
$$T_2 = W3 = 11.4 N$$
$$T_1 = 0.7036 T_2 = 8.0 N = W1$$

4. A 15 kg object rests on a table. A cord is attached to this object and also to a wall. Another object is hung from this cord as shown. If the coefficient of static friction between the 15 kg object and the table is 0.27, what is the maximum mass that can be hung, without movement?
   a) Draw the free body diagram
   b) Find the maximum mass that can be hung, without movement.

(***<u>VERY Important Note</u>*** – We can treat this system as a point particle rather than an extended object. Since all the *<u>tension</u>* <u>forces act at a single point</u> and the system is in equilibrium, each force produces NO torque and this is a just translational problem. You do not need to consider torque at all. You have to recognize how each tension force is related to other forces in the problem)



$$T_1 = f_s = \mu_s F_N$$
$$= 0.27(15)(9.8) = 39.69 N$$

$$\sum F_x = 0$$
$$-T_1 + T_{2x} = 0$$
$$39.69 = T_2 \cos 30$$
$$T_2 = 45.8 N$$

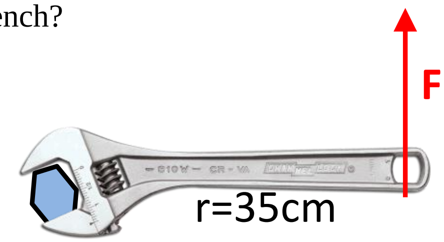
$$\sum F_y = 0$$
$$-W2 + T_{2y} = 0$$
$$W2 = T_2 \sin 30$$
$$mg = 45.8 \sin 30$$
$$m = 2.34 kg$$

Rotational and Translational Equilibrium Problems and Torque Problems

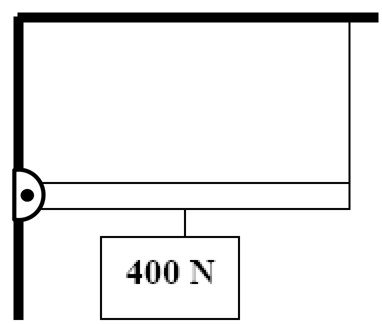
Use separate sheets of paper to solve these problems. Practice showing the steps of your work clearly, starting from a fundamental equation (Newtons 2nd law). For each problem draw a free body diagram of the extended object before solving. Remember that for the extended objects that we must now consider, the point of application of each force is important. Also remember that if an object is in rotational equilibrium, the net torque about ANY pivot point is zero. Therefore, you can choose your pivot point ANYWHERE - choose a point that is most convenient for solving the problem.

5. If the torque needed to loosen a lug nut is 45 Nm and you are using a 35 cm wheel wrench, what force do you need to exert perpendicular to the end of the wrench?

$$\tau = 45Nm$$
$$rF\sin 90 = 45Nm$$
$$0.35F = 45$$
$$F = 129N$$

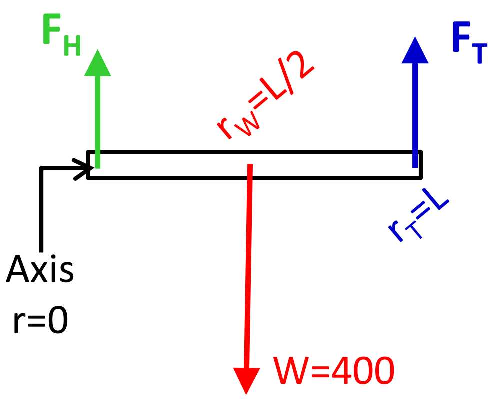


6. A beam of negligible mass is attached to a wall by a hinge. Attached to the center of the beam is a 400 N weight. A rope supports the beam as shown in the diagram. a) What is the tension in the rope?



The FBD should be of the forces on the beam only. We are not interested in the forces on the block or anything else. The forces act on the beam at different points and can therefore produce torque. Beam must be treated as an extended object and the forces in the free body diagram must come out of the appropriate location.

Since the beam is at rest, the net torque around ANY axis is zero. You can choose any axis or pivot point to solve the problem. Best to choose a pivot at a position with forces that are unknown such as the hinge.

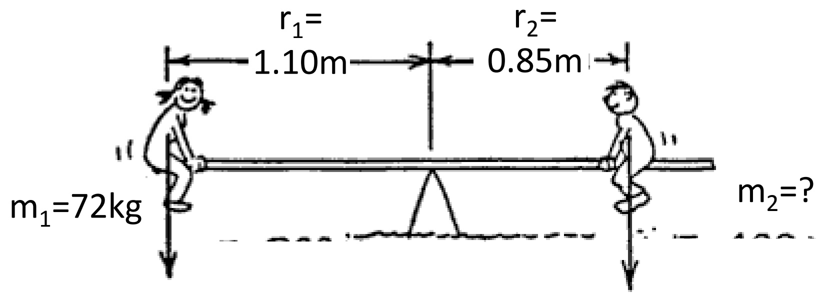


$$\sum \vec{\tau} = 0$$
$$\tau_T - \tau_W + \tau_{FH} = 0$$
$$F_T L\sin 90 - W(\tfrac{L}{2})\sin 90 + 0 = 0$$
$$F_T L = 400(\tfrac{L}{2})$$
$$F_T = 200N$$

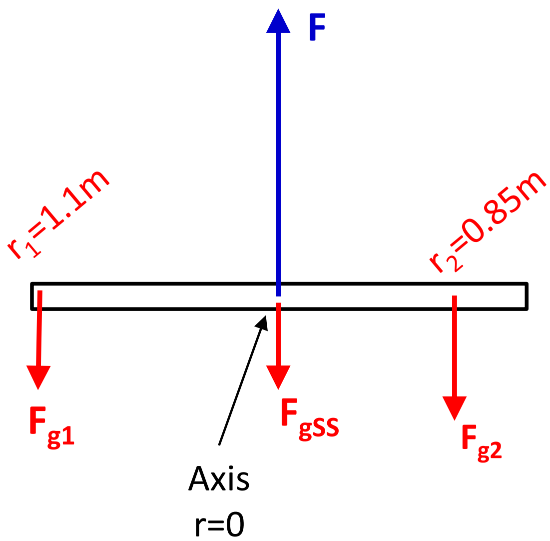
b) What is the force that the hinge exerts on the beam? We could either use the condition of translational equilibrium (Fnet = 0) OR set the rotation axis at the end of the beam where F_T is and sum the torques to zero to find F_H

$$\sum F_y = 0$$
$$-W + F_T + F_H = 0$$
$$F_H = 400 - 200 = 200N$$

7. Two students sit on either end of a uniform teeter-totter with the fulcrum located at the center. Student 1 sits 1.10 m from the pivot while Student 2 sits 0.85 m from the pivot. If Student 1 has a mass of 72 kg, what is the mass of Student 2?



The FBD should be of the forces on the seesaw only. We are not interested in the forces on the students or anything else. The forces act on the seesaw at different points and can therefore produce torque. seesaw must be treated as an extended object. Since the seesaw is at rest (in translational and rotational equilibrium), the net torque around ANY axis is zero. You can choose any axis or pivot point to solve the problem. Best to choose the pivot at the fulcrum since its support force is unknown and the weight of the see saw is unknown; if pivot pt is at fulcrum then forces that go through the pivot point produce no torque.



$$\sum \vec{\tau} = 0$$

$$\tau_1 - \tau_2 = 0$$

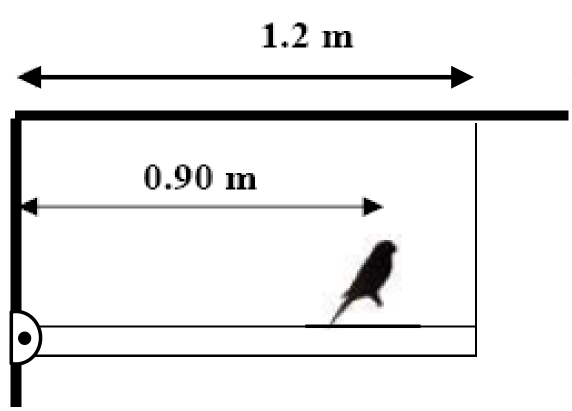
$$r_1 F_{g1} \sin 90 - r_2 F_{g2} \sin 90 = 0$$

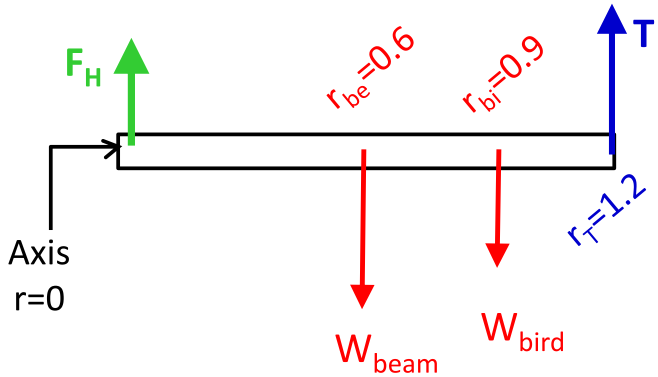
$$(1.10)(72g) - (0.85) m_2 g = 0$$

$$m_2 = \frac{(1.1)(72)}{0.85} = 93.2kg$$

8. A 0.75 kg bird stands on a uniform 1.0 kg stick as shown. The stick is attached to a wall with a hinge and to the ceiling with a rope of negligible mass. a) What is the tension in the rope?

The FBD should be of the forces on the beam only. We are not interested in the forces on the bird or the wall or anything else. Since the forces act on the beam at different points, beam must be treated as an extended object and the forces in the free body diagram must come out of the appropriate location. The weight of the beam acts at its center of gravity (the geometric center). Since the beam is at rest, the net torque around ANY axis is zero. You can choose any axis or pivot point to solve the problem. Best to choose a pivot at a position with forces that are unknown such as the hinge.





$$\sum \vec{\tau} = 0$$

$$\tau_T - \tau_{Bird} - \tau_{Beam} + \tau_F = 0$$

$$T(1.2)\sin 90 - W_{bird}(0.9)\sin 90 - W_{beam}(0.6)\sin 90 + 0 = 0$$

$$1.2T - 0.9(0.75)(g) - (0.6)(1)(g) = 0$$

$$1.2T - 6.615 - 5.88 = 0$$

$$T = 10.4N$$

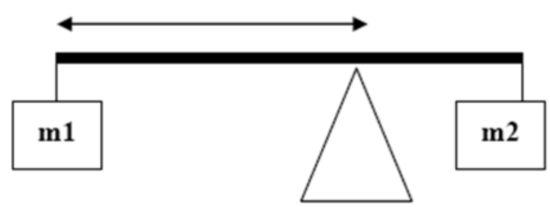
b) What is the force that the hinge exerts on the beam? We could either use the condition of translational equilibrium (Fnet = 0) OR set the rotation axis at the end of the beam where T is and sum the torques to zero to find the hinge force.

$$\sum \vec{F}_y = 0$$

$$F_H - W_{beam} - W_{bird} + T = 0$$

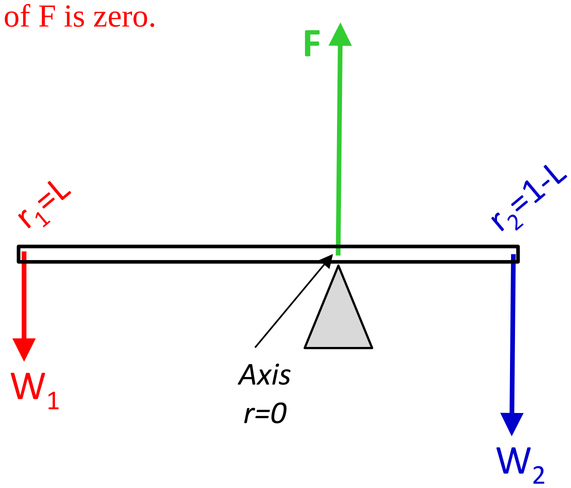
$$F_H = 1(9.8) + (0.75)(9.8) - 10.4 = 6.75N$$

9. Two masses (m1 = 3.00 kg, m2 = 5.00 kg) hang from the ends of a meter stick as shown. If the mass of the meter stick is negligible, at what distance from the left of the meter stick should a pivot be placed so that the system will be balanced?



The FBD should be of the forces on the meter stick only. We are not interested in the forces on the masses or anything else.

Since the beam is at rest, the net torque around ANY axis is zero. Because nothing is known about the support force of the fulcrum, best to place the rotation axis at the fulcrum so that the torque of F is zero.



$$\sum \vec{\tau} = 0$$

$$\tau_{W1} - \tau_{W2} = 0$$

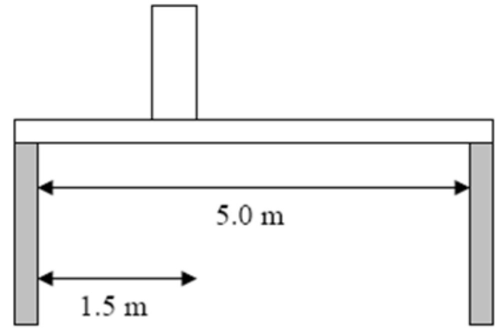
$$m_1 g L \sin 90 - m_2 g (1 - L) \sin 90 = 0$$

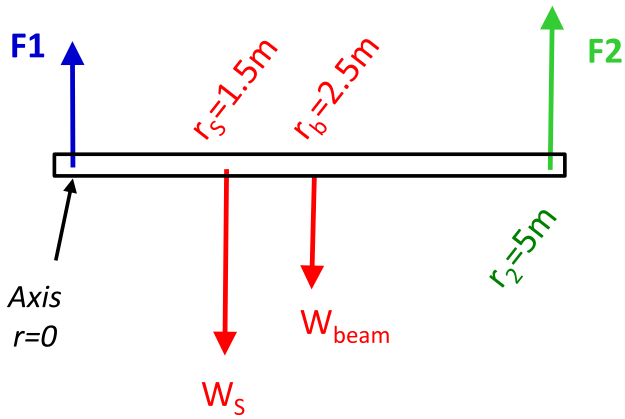
$$L(m_1 + m_2) - m_2 = 0$$

$$L = \frac{m_2}{m_1 + m_2} = \frac{5}{8} = 0.625m$$

10. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support:

The FBD should be of the forces on the beam only. We are not interested in the forces on the student or anything else. The weight of the beam acts at its center of gravity (the geometric center).





a) What is the force that the right support exerts on the beam?

$$\sum \vec{\tau} = 0$$

$$-\tau_{WS} - \tau_{Wbeam} + \tau_{F2} = 0$$

$$-1.5(650) - 2.5(250) + 5F2 = 0$$

$$F2 = 320N$$

b) W hat is the force that the left support exerts on the beam? We could either use the condition of translational equilibrium (Fnet = 0) OR set the rotation axis at the other end of the beam and sum the torques to zero to find F1.

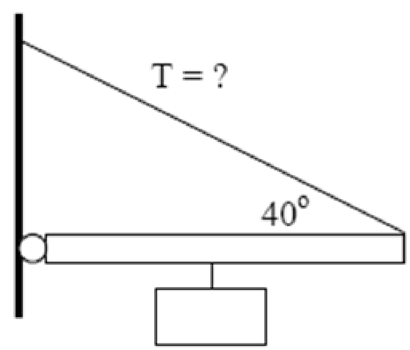
$$\sum \vec{F}_y = 0$$

$$F1 - W_S - W_{beam} + F2 = 0$$

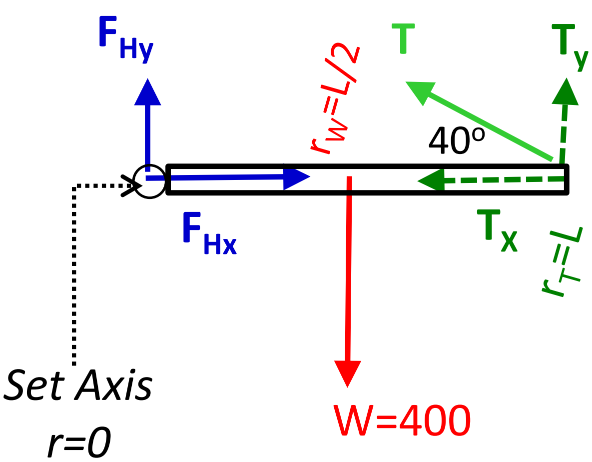
$$F1 = 650 + 250 - 320 = 580N$$

11. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the centre of the beam is a 400 N weight. A rope also helps to support this beam as shown in the diagram.

The FBD should be of the forces on the beam only. We are not interested in the force on the hanging mass or anything else.



a) What is the tension in the rope?



$$\sum \vec{\tau} = 0$$

$$-\tau_W + \tau_T = 0$$

$$-L/2(400)\sin 90 + LT\sin 40 = 0$$

$$-200 + 0.643T = 0$$

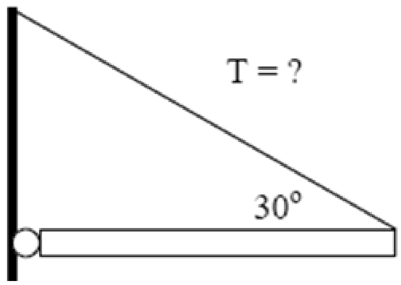
$$T = 311N$$

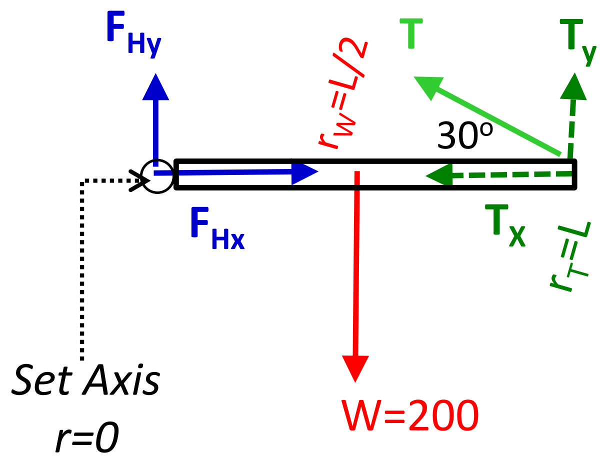
b) What are the vertical and horizontal forces that the wall exerts on the beam?

$$\sum \vec{F}_y = 0$$
$$F_{1y} - W + T_y = 0$$
$$F_{1y} = 400 - T\sin 40$$
$$= 200N$$

$$\sum \vec{F}_x = 0$$
$$F_{1x} - T_x = 0$$
$$F_{1x} = T\cos 40$$
$$= 238N$$

12. a) Find the tension in the rope supporting the 200 N hinged uniform beam as shown in the diagram.



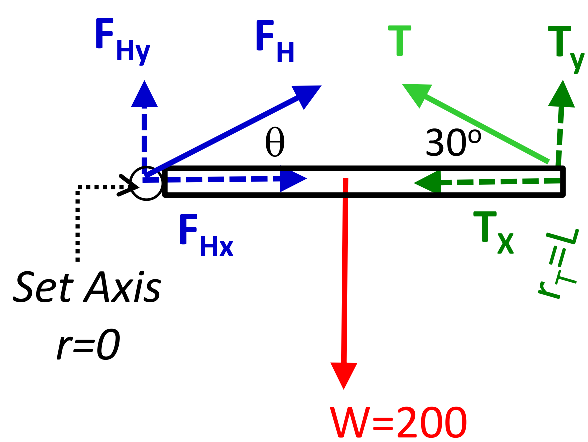


$$\sum \vec{\tau} = 0$$
$$-\tau_W + \tau_T = 0$$
$$-L/2(200)\sin 90 + LT\sin 30 = 0$$
$$-100 + 0.5T = 0$$
$$T = 200N$$

b) What are the magnitude and direction of the force that the hinge exerts on the beam?

$$\sum \vec{F}_y = 0$$
$$F_{1y} - W + T_y = 0$$
$$F_{1y} = 200 - 200\sin 30$$
$$F_{1y} = 100N$$

$$\sum \vec{F}_x = 0$$
$$F_{1x} - T_x = 0$$
$$F_{1x} = 200\cos 30$$
$$F_{1x} = 173.2N$$

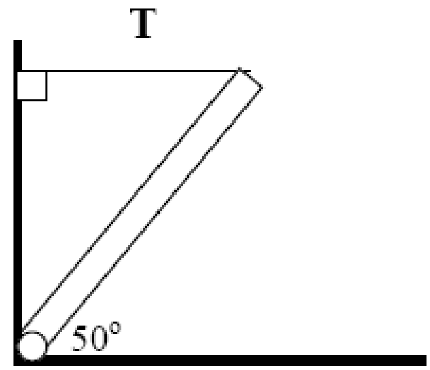


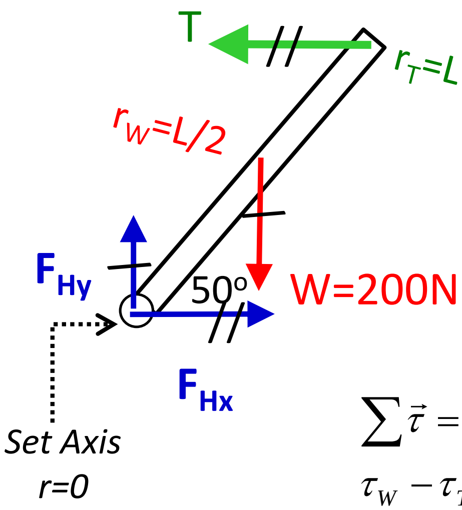
$$F_1^2 = F_{1x}^2 + F_{1y}^2$$
$$F_1 = \sqrt{100^2 + 173.2^2}$$
$$F_1 = 200N$$

$$\theta = \tan^{-1}\frac{F_{1y}}{F_{1x}} = 60^o$$

above horizontal

13. Find the tension in the rope supporting the 200 N hinged uniform beam as shown in the diagram.



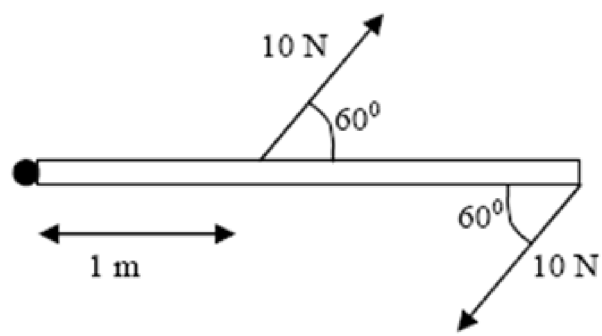


$$\sum \vec{\tau} = 0$$
$$\tau_W - \tau_T = 0$$
$$200(L/2)\sin 40 - TL\sin 50 = 0$$
$$64.28 + 0.766T = 0$$
$$T = 83.9N$$

14. The diagram below shows the top view of a door that is 2 m wide. Two forces are applied to the door as indicated in the diagram. What is the net torque on the door with respect to the hinge?

$$\sum \tau = \tau_1 - \tau_2$$
$$= (1)(10)\sin 60 - (2)(10)\sin 60$$
$$= 8.66 - 17.32$$
$$= -8.66Nm$$

So the net torque is CW



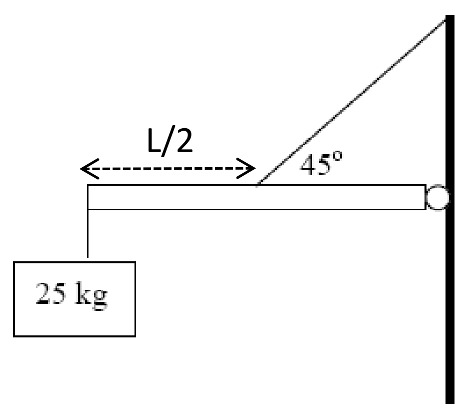
15. A uniform beam (mass = 22 kg) is supported by a cable that is attached to the center of the beam a shown in the diagram.

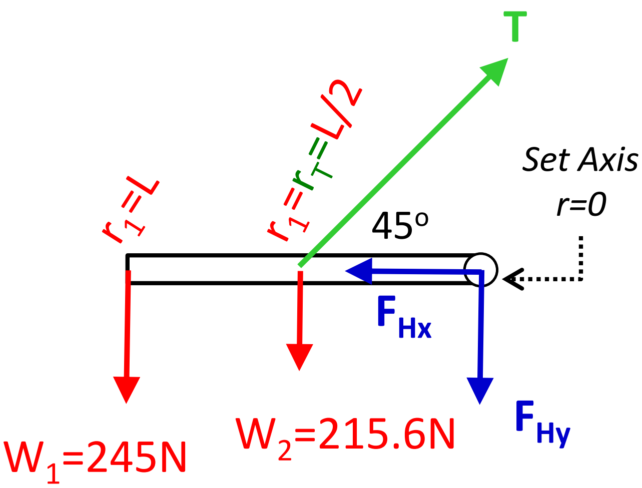
a) Find the tension in the cable.

In drawing the FBD,

- there has to be a downward hinge force since without it, there is an unbalanced torque (if pivot is set to center of beam, W1 would provide a single unbalanced torque without the vertical hinge torque)
- there has to be a negative horizontal hinge force since without it there is an unbalanced force (the x-comp of the tension)

<u>Note:</u> even if you have no idea what the forces provided by the hinge are, you can put the pivot there so that the hinge forces provide no torque. Then once the other forces are known, the hinge forces can be found using Fnet = 0.





$$\sum \vec{\tau} = 0$$
$$\tau_{W1} + \tau_{W2} - \tau_T = 0$$
$$W_1 L \sin 90 + W_2 (L/2) \sin 90 - T(L/2) \sin 45 = 0$$
$$245 + 107.8 - 0.354T = 0$$
$$T = 997N$$

b) find the horizontal and vertical forces acting on the hinge.

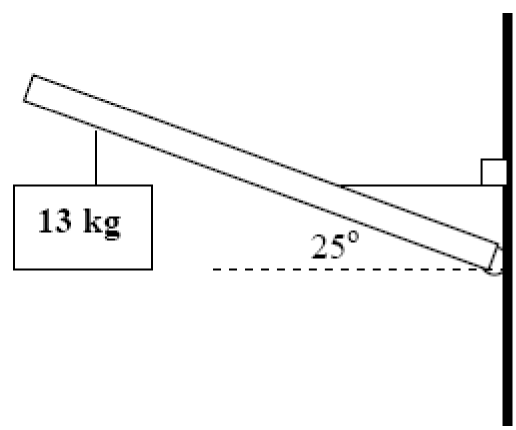
can find FHy using translational equilibrium (Fnety=0), but in this case, it is easier to just set the pivot to the center of the beam (where W2 and T would produce no torque) and use τ_{net}=0 (FHx produces no torque since it is in line with any pivot point on the beam)

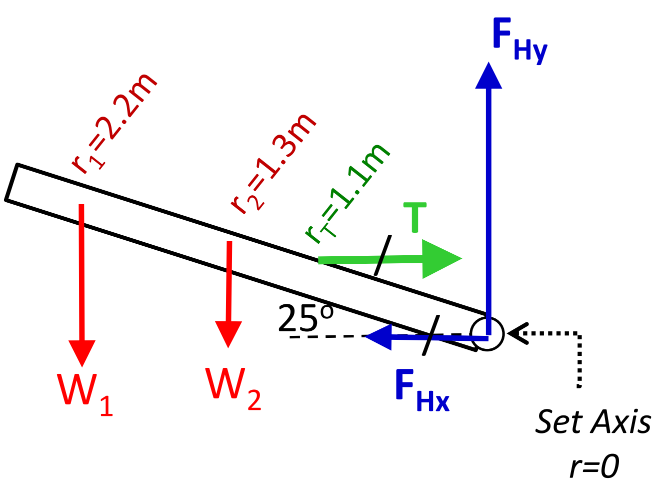
With pivot at center:

$$\sum \tau = 0$$
$$\tau_{W1} - \tau_{FH} = 0$$
$$W_1 (L/2) \sin 90 - F_{HY} (L/2) \sin 90 = 0$$
$$122.5 - F_{HY}/2 = 0$$
$$F_{Hy} = 245N$$

$$\sum \vec{F}_x = 0$$
$$T_x - F_{Hx} = 0$$
$$F_{Hx} = 997 \cos 45$$
$$F_{Hx} = 705N$$

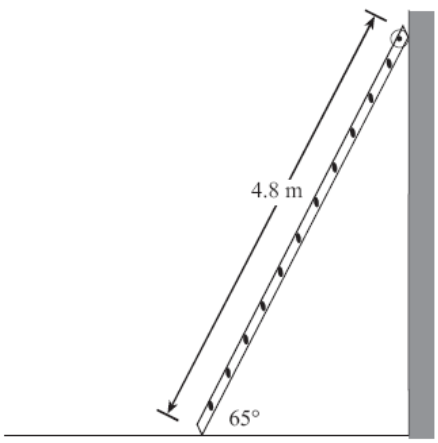
16. A 2.6 m uniform beam (mass of 9.0 kg) is attached to a wall by a hinge and supported by a rope. A 13 kg mass hangs from the beam 2.2 m from the hinge. Find the tension in the rope which is attached to the beam 1.1 m from the wall.

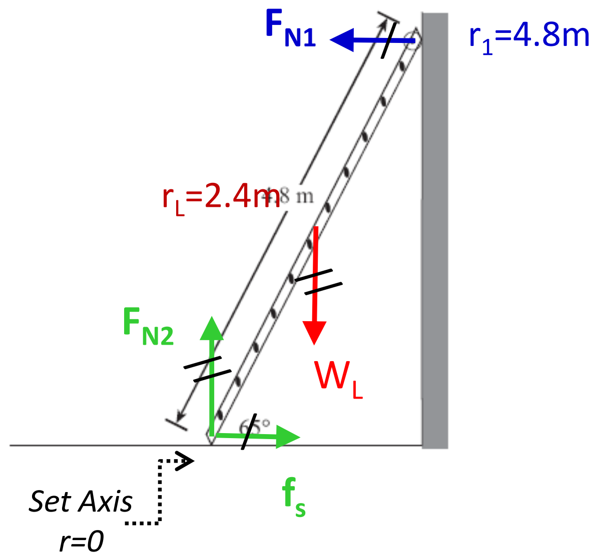




$$\sum \vec{\tau} = 0$$
$$\tau_{W1} + \tau_{W2} - \tau_T = 0$$
$$W_1 r_1 \sin 65 + W_2 r_2 \sin 65 - T r_T \sin 25 = 0$$
$$13g(2.2) \sin 65 + 9g(1.3) \sin 65 - T(1.1) \sin 25 = 0$$
$$254 + 103.92 - 0.465T = 0$$
$$T = 770N$$

17. A uniform 4.8 m long ladder of mass 16 kg leans against a frictionless vertical wall as shown in the diagram below. What minimum force of friction is needed at the base of the ladder to keep it from sliding?

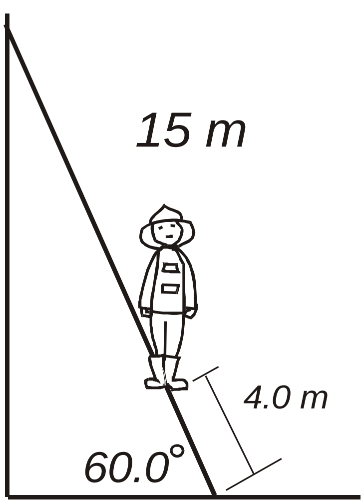


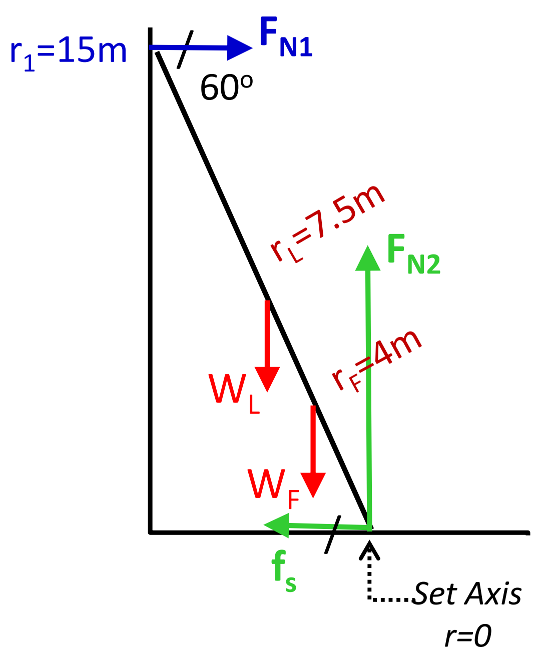


$$\sum \vec{\tau} = 0$$
$$-\tau_{WL} + \tau_{FN1} = 0$$
$$-(2.4)(16g)\sin 25 + 4.8F_{N1}\sin 65 = 0$$
$$-159 + 4.35F_{N1} = 0$$
$$F_{N1} = 36.6N$$

$$\sum F_x = 0$$
$$f_S - F_{N1} = 0$$
$$f_S = F_{N1} = 36.6N$$

18. A 15 m, 500.0 N uniform ladder rests against a frictionless wall. It makes 60.0° angle with the horizontal. Find (a) the horizontal and vertical forces on the base of the ladder if an 800.0 N fire fighter is standing 4.0 m from the bottom.



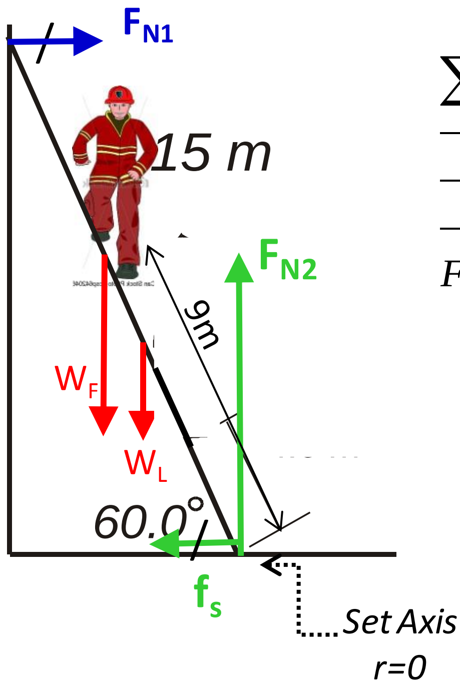


$$\sum \vec{\tau} = 0$$
$$-\tau_{FN1} + \tau_{WL} + \tau_{WF} = 0$$
$$-15F_{N1}\sin 60 + (7.5)(500)\sin 30 + (4)(800)\sin 30 = 0$$
$$-12.99F_{N1} + 1875 + 1600 = 0$$
$$F_{N1} = 268N$$

$$\sum \vec{F}_x = 0$$
$$f_S - F_{N1} = 0$$
$$f_S = F_{N1} = 268N$$

$$\sum \vec{F}_y = 0$$
$$-W_L - W_F + F_{N2} = 0$$
$$F_{N2} = 500 + 800 = 1300N$$

(b) If the ladder is on the verge of slipping when the fire fighter is 9.0 m from the bottom of the ladder, what is the coefficient of static friction on the bottom?



$$\sum \tau = 0$$
$$-\tau_{FN1} + \tau_{WL} + \tau_{WF} = 0$$
$$-15F_{N1}\sin 60 + (7.5)(500)\sin 30 + (9)(800)\sin 30 = 0$$
$$-12.99F_{N1} + 1875 + 3600 = 0$$
$$F_{N1} = f_s = 421N$$

$$f_s = \mu_s F_{N2}$$
$$421 = \mu_s(1300)$$
$$\mu_s = 0.32$$