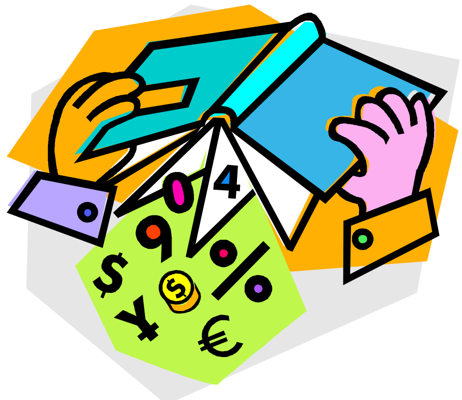


June 2012

Dear Geometry Students and Parents:

Welcome to Geometry! For the 2012-2013 school year, we would like to focus your attention to the fact that many concepts from Algebra I are infused into Geometry. In order to be successful in Geometry, a student must demonstrate a proficiency in certain skills including:

- Multiplying Polynomials
- Factoring
- Solving Systems of Equations
- Determining the Equations of Lines
    - ✓ Parallel Lines
    - ✓ Perpendicular Lines
- Simplifying Radicals

These topics will be reviewed briefly throughout the coming year; however, the topics are not re-taught in the Geometry course. To ensure that all students demonstrate the basic algebra skills to be successful, an assessment will be administered within the first **two weeks** of school (date to be determined). The attached review packet is provided for practice and is intended as a tool for assessment readiness. A one-day, full class period review will be conducted in each class. In addition, students are encouraged to seek extra help before or after school from their teacher for any topics requiring more personal in-depth remediation. Additional practice problems and explanations may also be found beginning on page 869 of the Geometry textbook.

It is expected that each student will fully complete the review packet. Packets will be collected in class on the day of the assessment. If you have any questions, please do not hesitate to contact your child's teacher.

Mr. Hyman
Ms. Jewett
Ms. Kast
Ms. Marchegiano
Ms. Riggi
Mrs. Poyner
Ms. Shulman
Mr. Waddon

Binomial Products
To multiply two binomials, you can use the Distributive Property systematically. You may know this as the "**FOIL**" method or "**Area Model**".

Example: Find the product

Simplify $(x+2)(3x-4)$

$= x(3x) + x(-4) + 2(3x) + 2(-4)$ Distribute

$= 3x^2 - 4x + 6x - 8$ Simplify--multiply

$= 3x^2 + 2x - 8$ Simplify--combine like terms

Simplify $(7a-1)^2$

$= (7a-1)(7a-1)$ Rewrite as the product of the binomial and itself

$= 7a(7a) + 7a(-1) + (-1)(7a) + (-1)(-1)$ Distribute

$= 49a^2 - 7a - 7a + 1$ Simplify--multiply

$= 49a^2 - 14a + 1$ Simplify--combine like terms

Simplify

1) $(x-1)^2$

2) $(3x+4)^2$

3) $(-9a^2-2x)^2$

Factoring

Factoring (Common Monomial Factor)

Solution:

- The first step in ANY factoring problem is to factor out the GCF. The other factor is the original polynomial divided by the GCF.
- A variable must be in each term of the polynomial in order to be in the GCF. Always use the smallest exponent in the GCF.
- If there is no common factor, the polynomial can be "**prime**."

Example: Factor completely $24xy + 18xy^2 - 3y$

$24xy + 18xy^2 - 3y$	Initial polynomial
GCF = $3y$	Find the GCF
$\dfrac{24xy + 18xy^2 - 3y}{3y} = 8x + 6xy - 1$	Determine other factor by dividing each term by the GCF
$3y(8x + 6xy - 1)$	Final factorization
	Look to see if the polynomial factors further.

<u>Factor Completely</u>

4) $3y^2 - 9y$

5) $54x^5y^3z^2 - 9x^2y^6r^3 + 36x^4r^7$

6) $5x^2y^2 - 9r^3 + 7z$

Factoring (trinomial where $a = 1$)

Solution:

- Put expression into standard form, if not already done.
  Standard form: $ax^2 + bx + c$
- If necessary, factor out the GCF or a "−1" (if the a is negative). Then look to see if you can factor further.
- Look to see if the resulting trinomial can be factored further. For any trinomial of form $x^2 + bx + c$, it will factor if there are two factors of c that add up to b (i.e., factors f_1 and f_2, such that $f_1 \cdot f_2 = c$ and $f_1 + f_2 = b$). The trinomial can then be factored as $(x + f_1)(x + f_2)$
- When finding the factors f_1 and f_2, remember that if c is negative, f_1 and f_2 must have different signs (one positive and one negative). If c is positive, then f_1 and f_2 must both have the same sign as b.
- If there are *no two such factors*, then the trinomial is "**prime**."

Example: Factor completely $2x^2 - 20x + 42$

$2x^2 - 20x + 42$	Standard Form, if necessary.
$2(x^2 - 10x + 21)$	Factor out the GCF of 2.
$f_1 = -7$, $f_2 = -3$	Find the factors of +21 that add up to -10. They must both be negative.
$2(x - 7)(x - 3)$	Use f_1 and f_2 to factor the trinomial. *As part of the final answer, **don't** forget to bring down the G.C.F.*

Factor Completely

7) $x^2 - 5x + 6$

8) $x^2 + 23x + 42$

9) $x^2 - 10x + 24$

10) $-7x + x^2 + 20$

11) $3x^2 + 15x + 18$

12) $-x^2 - 5x + 50$

Factoring (trinomial where $a \neq 1$)

Solution:

There are several methods for factoring these types of trinomials. If you prefer another method, see your teacher if you have any questions.

Example: *Factor completely* $-5x-6+6x^2$

Method 1: AC Method

$-5x-6+6x^2$	Initial trinomial
$6x^2-5x-6$	**Step 1:** Put into standard form, if necessary
not necessary	**Step 2:** Factor out GCF or −1, if necessary
$(6)(-6)=-36$	**Step 3:** Multiply $a \cdot c$
$f_1=4$, $f_2=-9$	**Step 4:** Find the factors of the product of $a \cdot c$ that add up to b (i.e., factors f_1 and f_2, such that $f_1 \cdot f_2 = a \cdot c$ and $f_1 + f_2 = b$). Remember that if $a \cdot c$ is negative, f_1 and f_2 must have different signs (one positive and one negative). If $a \cdot c$ is positive, then f_1 and f_2 must both have the same sign as b.
$6x^2-5x-6$ $=6x^2+4x-9x-6$	**Step 5:** Rewrite the middle term as the sum of two terms whose coefficients are the numbers above.
$=2x(3x+2)-3(3x+2)$ ** $=(3x+2)(2x-3)$	**Step 6:** Factor by grouping **the expression in the parentheses must be the same

Method 2: Slide-Divide-Slide

$6x^2 - 5x - 6$	Initial trinomial.
not necessary	**Step 1:** Factor out GCF or −1, if necessary.
$6x^2 - 5x - 6$ $(6)(-6) = -36$	**Step 2:** Multiply $a \cdot c$. **(SLIDE)**
$x^2 - 5x - 36$	**Step 3:** Leading coefficient becomes 1 and c is replaced with $a \cdot c$. **if this trinomial is not factorable, then the original trinomial is prime
$f_1 = 4$, $f_2 = -9$	**Step 4:** Find the factors of −36 that add up to −5
$(x+4)(x-9)$	**Step 5:** Use $f_1 and\ f_2$ to factor the trinomial.
$\left(x+\frac{4}{6}\right)\left(x-\frac{9}{6}\right)$	**Step 6: DIVIDE** $f_1 and\ f_2$ by “a”.
$\left(x+\frac{2}{3}\right)\left(x-\frac{3}{2}\right)$	**Step 7:** Simplify fractions, if necessary.
$(3x+2)(2x-3)$	**Step 8: SLIDE** denominators of each factor to become the coefficient of x.

Factor completely using the method of your choice.

13) $2x^2 - 11x + 5$

14) $8x^2 + 6x - 9$

15) $-8x^2 + 6x + 2$

Factoring (binomial)

Solution: $a^2 - b^2 = (a+b)(a-b)$

"Difference of Two Perfect Squares"

- Must be a binomial (two terms)
- Terms must be subtracted
- Each term must be a perfect square

A binomial with the ***sum* of two perfect squares is **prime**.

Example: Factor completely $81x^2 - 16$

$81x^2 - 16$	Original binomial
not necessary	Factor out G.C.F. or −1, if necessary
$81x^2 - 16$	Are the terms subtracted? YES!
$81x^2 = (9x)^2$ $16 = (4)^2$	Are the first and last terms perfect squares? YES!
$81x^2 - 16$ $= (9x+4)(9x-4)$	Answer is the product of the sum and difference of the square roots.

<u>Factor completely</u>

16) $25x^2 - 1$

17) $100 - 36y^2$

18) $64y^4 + 49x^6$

Solving Quadratics by Factoring

Solution:

- Re-write the quadratic in standard form $ax^2 + bx + c = 0$.
- Factor the polynomial completely
- Set each factor equal to zero (use "or" between the equations)
- Solve each resulting equation to obtain ***two*** solutions.

Example: Solve $6x^2 - 5x = 6$

$6x^2 - 5x = 6$	Original equation
$6x^2 - 5x - 6 = 0$	Write in standard form
$(2x-3)(3x+2) = 0$	Factor the polynomial completely
$2x - 3 = 0$ **or** $3x + 2 = 0$	Set each factor equal to zero
$x = \frac{3}{2}$ **or** $x = -\frac{2}{3}$	Solve both equations

Tip: If you have a negative sign in front of the x^2 term, simply multiply both sides of the equation by -1, then go ahead and factor. Similarly, divide both sides by any GCF. For example, change $-18x^2 + 15x + 18 = 0$ to $6x^2 - 5x - 6 = 0$ by dividing both sides by -3.

Solve

19) $-16 = x^2 + 10x$

20) $-2x^2 + 70 = -4x$

21) $24x^2 + 10x - 6 = 0$

Graphing

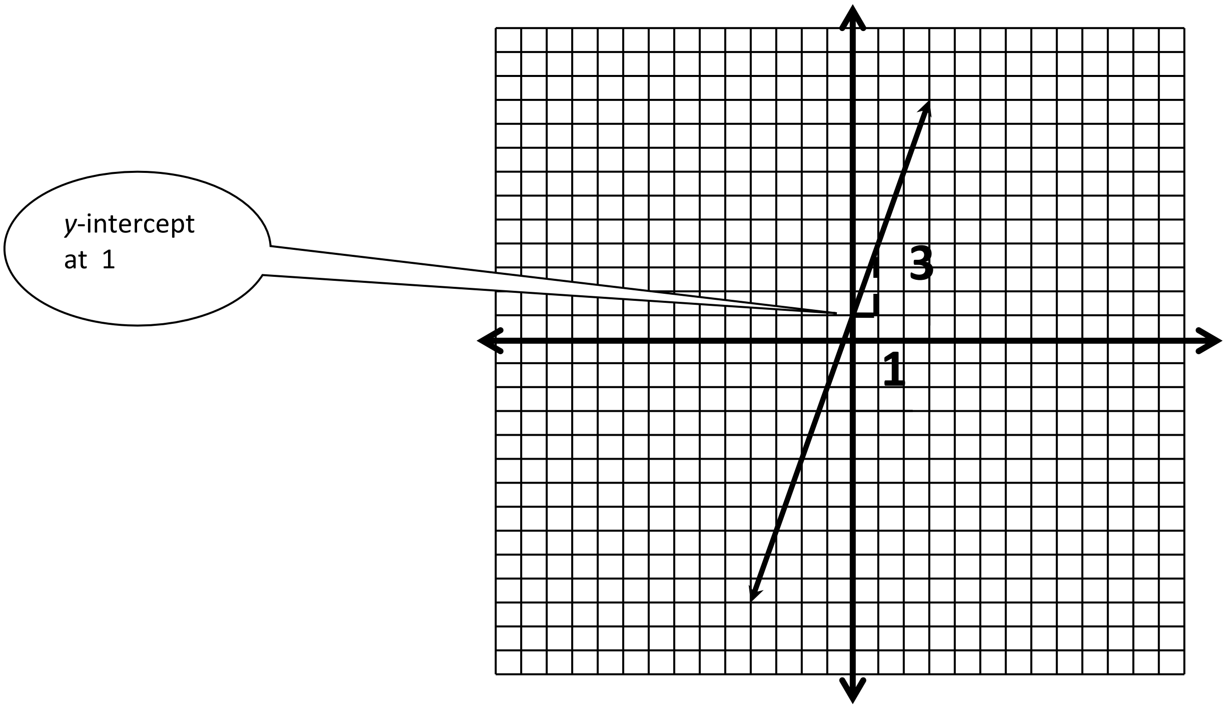
Determine and graph the equation of a line $y = mx + b$

Solution:

- Determine slope from the two given points using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Determine the y-intercept (b) by using $y = mx + b$
  - ✓ substitute a point for the x and y variables
  - ✓ substitute the slope for the m variable
  - ✓ solve for b
- Rewrite the equation $y = mx + b$ by replacing the m and b variables with the values you found.
- Use the slope and y-intercept to graph.

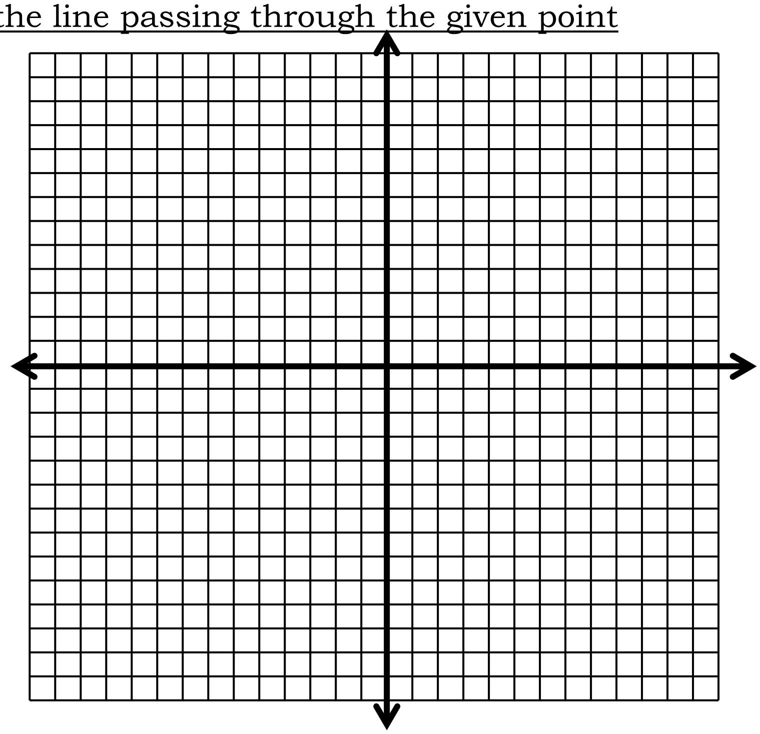
Example: Determine and graph the equation of the line passing through the points $(2, 7)$ and $(-1, -2)$.

$m = \frac{7-(-2)}{2-(-1)} = \frac{9}{3} = 3$ $m = \frac{3}{1}$ or $\frac{-3}{-1}$	Determine slope
$y = mx + b$ $7 = (3)(2) + b$ $7 = 6 + b$ $1 = b$	Determine the y-intercept —you may use either point
$y = 3x + 1$	Write the equation
Graph on the next page	Graph equation using y-intercept and slope

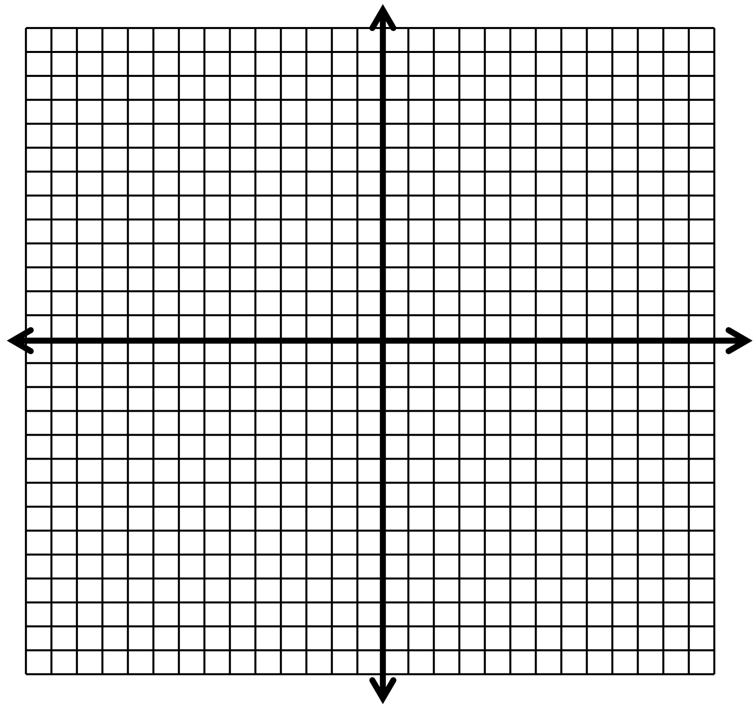


Determine and graph the equation of the line passing through the given point

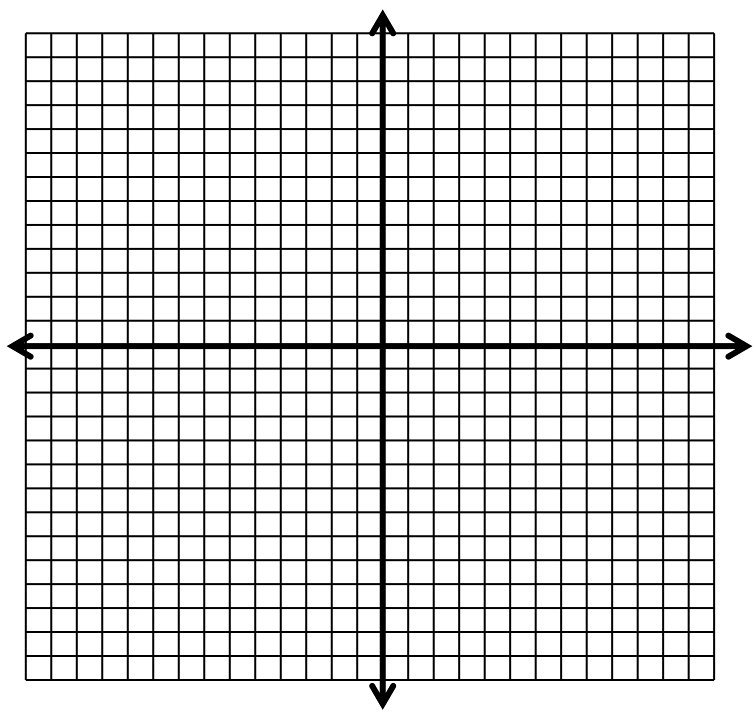
22) (0, 7) and (1, 9)



23) $(3,\ -1)$ and $(6,\ 7)$



24) $(-2,\ -6)$ and $(-2,\ 4)$



Determine the equation of a line parallel or perpendicular to another line.

Example: Determine the equation of a line in slope-intercept form which is perpendicular/parallel to the line $2x-3y=8$ and passes through the point $(-1,\ 5)$.

Solution:

Step 1: Determine the slope of the given equation by writing in slope-intercept form.

$$2x-3y=8$$
$$-3y=-2x+8$$
$$y=\frac{2}{3}x-\frac{8}{3}$$

The slope is $\frac{2}{3}$

Step 2: Determine the slope of the perpendicular line and parallel line

$$m_{\perp}=-\frac{3}{2}$$

The slopes of perpendicular lines are opposite reciprocals

$$m_{\parallel}=\frac{2}{3}$$

The slopes of parallel lines are equal

Step 3: Using the slope from step 2 and the given point, determine the y-intercept (b).

Perpendicular

$$y=mx+b$$
$$5=\left(-\frac{3}{2}\right)(-1)+b$$
$$5=\frac{3}{2}+b$$
$$\frac{7}{2}=b$$

Parallel

$$y=mx+b$$
$$5=\left(\frac{2}{3}\right)(-1)+b$$
$$5=-\frac{2}{3}+b$$
$$\frac{17}{3}=b$$

Step 4: Write the equation of the line by substituting your values for m and b.

Perpendicular

$$y=-\frac{3}{2}x+\frac{7}{2}$$

Parallel

$$y=\frac{2}{3}x+\frac{17}{3}$$

25) Determine the equation of a line in slope-intercept form which is perpendicular to the line $x+4y=8$ and passes through the point $(2, -6)$.

26) Determine the equation of a line in slope-intercept form which is parallel to the line $-4x-3y=-1$ and passes through the point $(-3, -3)$.

27) Determine the equation of a line in slope-intercept form which is perpendicular to the line $-4x+2y=8$ and passes through the point $(1, 6)$.

System of Equations

Solution:

There are three ways to solve a system of equations:

1. Graphing Method
2. Substitution Method
3. Elimination Method

We will demonstrate the substitution and elimination methods.

Example: Consider the system of equations: $\begin{cases} \mathbf{-2x + y = 4} \\ \mathbf{x + y = -5} \end{cases}$

<u>Substitution method</u>:

Step 1: Solve for ***one*** variable in ***one*** of the equations.

Step 2: Substitute the expression into the **<u>OTHER</u>** equation.

Step 3: Solve your new equation.

Step 4: Substitute the value of variable into one of the original equations to determine second variable.

Step 5: Write your answer as an ordered pair.

Step 1: Solve for one variable

$\mathbf{-2x + y = 4}$

$\mathbf{y = 2x + 4}$

Substitute

$\mathbf{x + (2x + 4) = -5}$

$\mathbf{3x + 4 = -5}$

$\mathbf{3x = -9}$

$\mathbf{x = -3}$

Substitute

$\mathbf{-3 + y = -5}$

$\mathbf{y = -2}$

Solution: $(-3, -2)$

Elimination Method:

Step 1: Line up the variables on one side and the constants on the other side.

Step 2: Determine a variable you want to eliminate—you want the coefficients to be the same number with opposite signs.

Step 2: If necessary, multiply one or both equations by a number to force the coefficients to be the same number with opposite signs.

Step 3: **Add** the equations together so that one variable cancels out.

Step 4: Solve the equation.

Step 5: Substitute the value of variable into one of the original equations to determine the value of the second variable

Step 5: Write your answer as an ordered pair.

Eliminate the x variable by multiplying the second equation by 2

$$\begin{cases} -2x + y = 4 \\ x + y = -5 \end{cases}$$

$$\begin{aligned} -2x + y &= 4 \\ 2(x + y &= -5) \end{aligned} \quad \Rightarrow \quad \begin{array}{r} \cancel{-2x} + y = 4 \\ + \ \underline{\cancel{2x} + 2y = -10} \\ 3y = -6 \\ y = -2 \end{array}$$

Substitute

$$-2x + (-2) = 4$$

$$-2x = 6$$

$$x = -3$$

Solution: $(-3, -2)$

Solve the following system of equations by substitution.

28) $\begin{cases} x + 2y = 12 \\ x - y = -3 \end{cases}$

29) $\begin{cases} -4x + 3y = -19 \\ 2x + y = 7 \end{cases}$

Solve the following system of equations by elimination.

30) $\begin{cases} 2x - 3y = 8 \\ 6x - 9y = 12 \end{cases}$

31) $\begin{cases} 8x - 5y = 14 \\ 10x - 2y = 9 \end{cases}$

Radicals

Simplify Radicals

Radic
al

$$\sqrt[index]{radicand}$$

Solutions:

- You will need to know all your perfect squares $1^2 = 1$ through $20^2 = 400$.
- To simplify $\sqrt{\text{radicand}}$

$$= \sqrt{\text{perfect square}\cdot\text{non-perfect square}}$$

$$= \sqrt{\text{perfect square}} \cdot \sqrt{\text{non-perfect square}}$$

$$= \text{square root of perfect square}\sqrt{\text{non-perfect square}}$$

Examples: Simplify the following

1. $\sqrt{121}$
   $= 11$

2) $\sqrt{48}$
   $= \sqrt{16 \cdot 3}$
   $= 4\sqrt{3}$

3) $\sqrt{-12}$
   =non-real answer

4) $-5\sqrt{128}$
   $= -5\sqrt{64 \cdot 2}$
   $= -5 \cdot \sqrt{64} \cdot \sqrt{2}$
   $= -5 \cdot 8 \cdot \sqrt{2}$
   $= -40\sqrt{2}$

Add/Subtract Radicals

Solution:

- In order to add or subtract radicals, the index and the radicand must be the same!
- You combine radicals as if you were combining like-terms—add/subtract their coefficients **ONLY**.
- Sometimes it may be necessary to simplify the radicals before you add or subtract.

Examples: Simplify. Express your answer in simplified radical form.

1) $5\sqrt{13} - 8\sqrt{13}$

$= -3\sqrt{13}$

2) $-\sqrt{99} + 3\sqrt{11}$

$= -\sqrt{9 \cdot 11} + 3\sqrt{11}$

$= -\sqrt{9} \cdot \sqrt{11} + 3\sqrt{11}$

$= -3\sqrt{11} + 3\sqrt{11}$

$= 0$

3) $\sqrt{175} - 2\sqrt{28}$

$= \sqrt{25 \cdot 7} - 2\sqrt{4 \cdot 7}$

$= \sqrt{25} \cdot \sqrt{7} - 2 \cdot \sqrt{4} \cdot \sqrt{7}$

$= 5\sqrt{7} - 2 \cdot 2\sqrt{7}$

$= 5\sqrt{7} - 4\sqrt{7}$

$= \sqrt{7}$

Multiplication/Division of Radicals

Solution: **This is NOT the only way to simplify these problems**

- Simplify the radical first.
- For multiplication:
  - ✓ Multiply numbers outside the radical together
  - ✓ Multiply numbers under the radical together
  - ✓ Make sure to check if your radical needs to be simplified again
- For division:
  - ✓ Divide numbers outside the radical
  - ✓ Divide numbers under the radical
  - ✓ Make sure to check if your radical needs to be simplified again

Examples: Simplify. Express your answer in simplified radical form.

1) $\frac{\sqrt{192}}{\sqrt{3}} = \frac{\sqrt{64\cdot 3}}{\sqrt{3}} = \frac{\sqrt{64}\cdot\sqrt{3}}{\sqrt{3}} = \frac{8\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = 8$

2) $\sqrt{\frac{2}{49}} = \frac{\sqrt{2}}{7}$

3) $2\sqrt{6}\cdot -\sqrt{27}$

$= 2\sqrt{6}\cdot -\sqrt{9\cdot 3}$

$= 2\sqrt{6}\cdot -\sqrt{9}\cdot\sqrt{3}$

$= 2\sqrt{6}\cdot -3\sqrt{3}$

$= (2)\cdot(-3)(\sqrt{6})(\sqrt{3})$

$= -6\sqrt{18}$

Don't forget to simplify the

$= -6\sqrt{9\cdot 2}$

$= -6\sqrt{9}\cdot\sqrt{2}$

$= -6\cdot 3\cdot\sqrt{2}$

$= -18\sqrt{2}$

Simplify. Express your answer in simplified radical form.

32) $-\sqrt{169}$

33) $\sqrt{80} - 14\sqrt{5}$

34) $-3\sqrt{2} \cdot \sqrt{50}$

35) $\frac{\sqrt{120}}{\sqrt{8}}$

36) $\sqrt{\frac{72}{9}}$

37) $5\sqrt{2} + 2\sqrt{128}$

Algebra 1 Review Packet Answer Sheet

1) $x^2 - 2x + 1$

2) $9x^2 + 24x + 16$

3) $81a^4 + 36a^2x + 4x^2$

4) $3y(y - 3)$

5) $9x^2(6x^3y^3z^2 - y^6r^3 + 4x^2r^7)$

6) prime

7) $(x - 2)(x - 3)$

8) $(x + 2)(x + 21)$

9) $(x - 4)(x - 6)$

10) prime

11) $3(x + 3)(x + 2)$

12) $-(x + 10)(x - 5)$

13) $(x - 5)(2x - 1)$

14) $(2x + 3)(4x - 3)$

15) $-2(x - 1)(4x + 1)$

16) $(5x + 1)(5x - 1)$

17) $4(5 + 3y)(5 - 3y)$ or $-4(3y - 5)(3y + 5)$

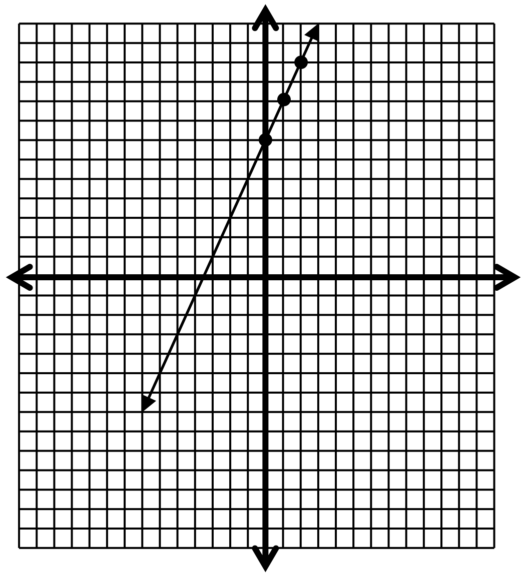
18) prime

19) $x = -8$ or $x = -2$

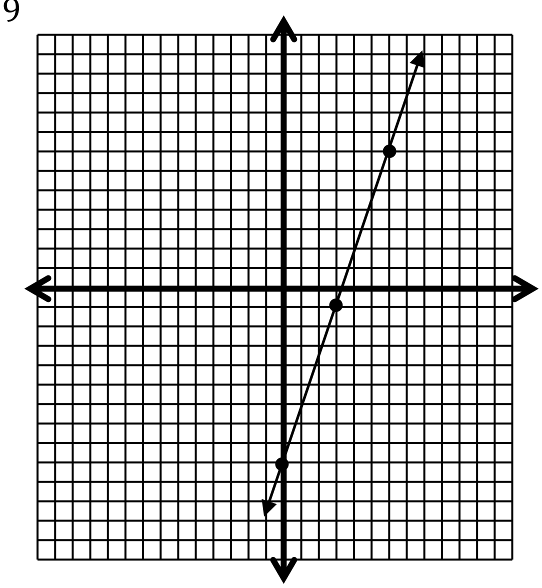
20) $x = 7$ or $x = -5$

21) $x = -\frac{3}{4}$ or $x = \frac{1}{3}$

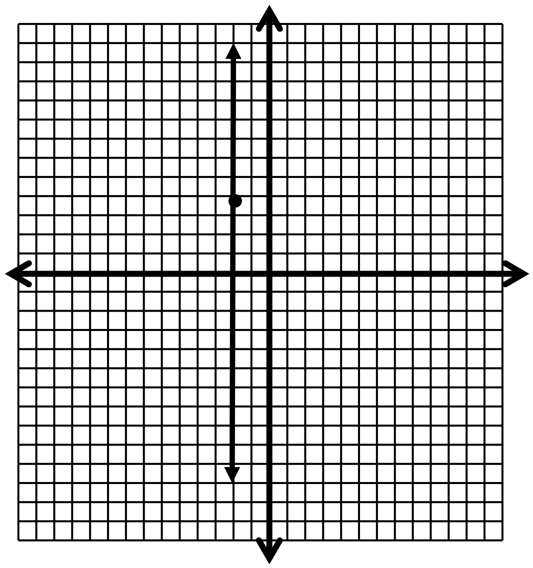
22) $y = 2x + 7$



23) $y = \frac{8}{3}x - 9$



24) $x = -2$



25) $y = 4x - 14$

26) $y = -\frac{4}{3}x - 7$

27) $y = -\frac{1}{2}x + \frac{13}{2}$

28) $(2, 5)$

29) $(4, -1)$

30) $\emptyset$ or No Solution

31) $(\frac{1}{2}, -2)$

32) -13

33) $-10\sqrt{5}$

34) -30

35) $\sqrt{15}$

36) $2\sqrt{2}$

37) $21\sqrt{2}$