Radial Net Forces and Circular Motion Problems (#3)

For each of the problems below, <u>a diagram is really important</u>. You must begin your solution with a clear, accurate free body diagram. Show your solutions step by step starting with the basic conceptual equation (Newton's 2nd Law). Use separate pieces of paper to solve these problems.

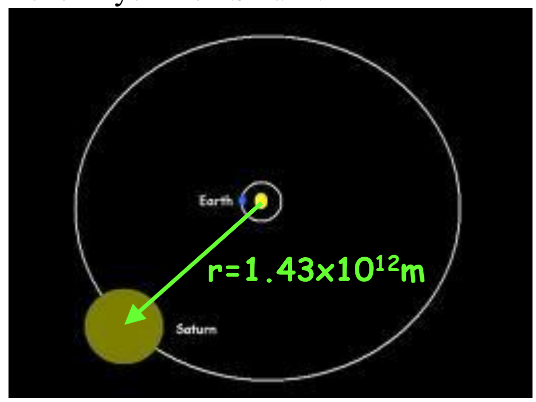
1. Saturn travels at an average speed of 9.66×10^3 m/s around the Sun in a roughly circular orbit. Its distance from the Sun is 1.43×10^{12} m. How long in seconds and earth years is a "year" on Saturn?

$$v = \frac{2\pi r}{T}$$

$$9660 = \frac{2\pi(1.43x10^{12})}{T}$$

$$T = 9.30x10^8 s = 29.5 yrs$$

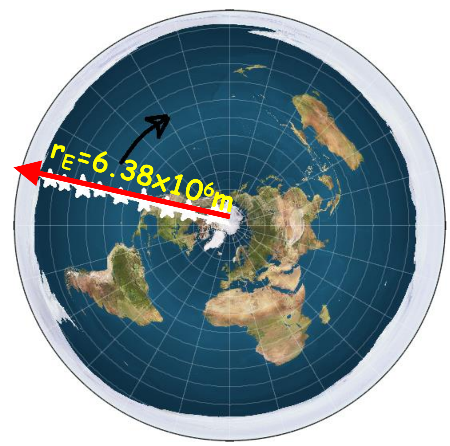
V=9660m/s



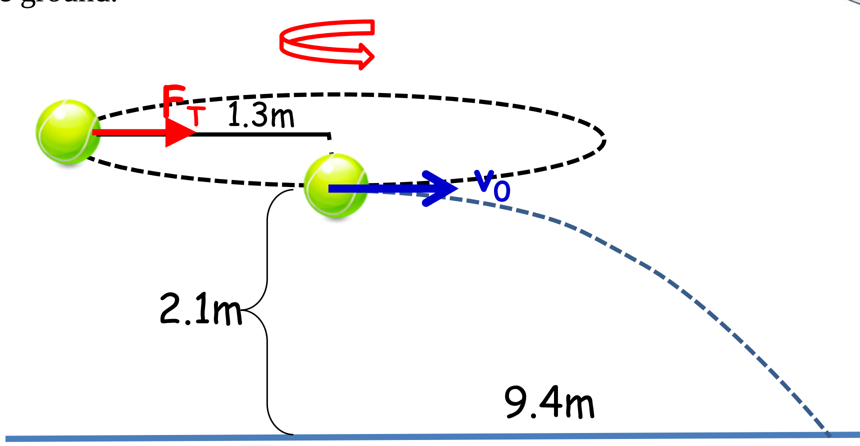
2. Consider the radius of the Earth to be 6.38×10^6 m. What is the magnitude of the centripetal acceleration experienced by a person (a) at the equator and (b) at the north pole due to the Earth's rotation?

$$a_C = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

a) At equator $a_c = 0.034$ m/s^2 = 0.0034 g's
b) At pole, r = 0 so $a_c = 0$



3. You tie a string to a rock and twirl it at a constant speed in a horizontal circle with a radius of 1.30 m, 2.10 m above the ground. The rock comes loose and travels as a projectile a horizontal distance of 9.40 m before striking the ground.



(a) What was the magnitude of the centripetal acceleration of the rock when it was on the string?
(b) What was the speed when it was on the string?

$$a_C = \frac{v^2}{r}$$

Need to know v_0. Can find v_0 from the projectile motion, since the projectile is launched with the initial horizontal velocity equal to the tangential velocity of the uniform circular motion.

Projectile Motion

$v_x = ?$

$\Delta x = 9.4m$

t

$v_{0y} = 0$

v_{fy}

$a_y = -9.8m/s^2$

$\Delta y = -2.1m$

t

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$-2.1 = \tfrac{1}{2}(-9.8)t^2$$

$$t = 0.655s$$

$$\Delta x = v_x t$$

$$9.4 = v_x(0.655)$$

$$v_x = 14.35m/s$$

Circular Motion

$$a_C = \frac{v^2}{r} = \frac{14.35^2}{1.3} = 158.4m/s^2$$

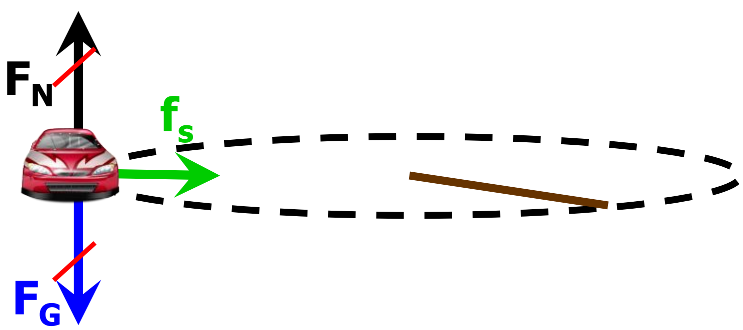
4. An astronaut in training rides in a seat that is moved in uniform circular motion by a radial arm 5.10 m long. If her speed is 15.0 m/s, what is the centripetal force on her in G's, where one G equals her weight on earth?

$$F_C = \frac{mv^2}{r} = \frac{m(15^2)}{5.1} = 44.1m = 4.5mg \quad 4.5G's$$

OR

$$a_C = \frac{v^2}{r} = \frac{15^2}{5.1} = 44.1 = 4.5g$$

5. Fifteen clowns are late to a party. They jump into their sporty coupe and start driving. Eventually they come to a level curve, with a radius of 27.5 m. What is the top speed at which they can successfully drive around the curve? The coefficient of static friction between the car's tires and the road is 0.800.



$$F_c = \sum F_r = ma_c$$

$$f_S = \frac{mv^2}{r}$$

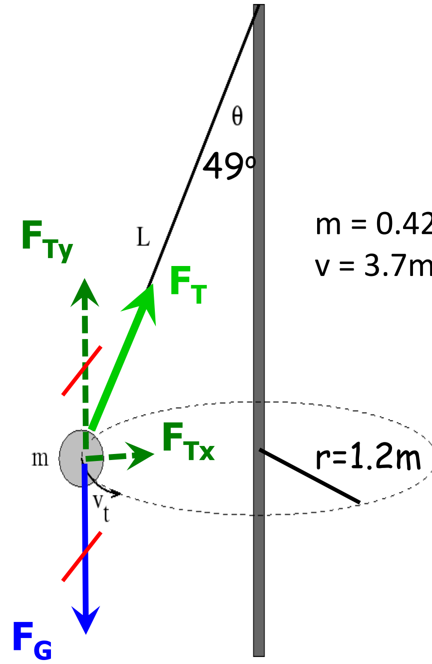
$$\mu_S F_N = \frac{mv^2}{r}$$

$$\mu_S mg = \frac{mv^2}{r}$$

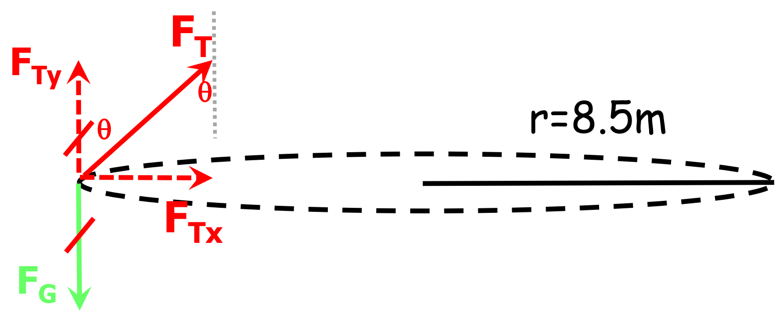
$$v = \sqrt{\mu_S gr} = 14.7m/s$$

6. You are playing tetherball with a friend and hit the ball so that it begins to travel in a circular horizontal path. If the ball is 1.2 meters from the pole, has a speed of 3.7 m/s, a mass of 0.42 kg, and its (weightless) rope makes a 49^0 angle with the pole, find the tension force that the rope exerts on the ball just after you hit it.

$$F_c = \sum F_r = ma_c$$
$$F_{Tx} = \frac{mv^2}{r}$$
$$F_T \sin 49 = \frac{mv^2}{r}$$
$$F_T = \frac{mv^2}{r \sin 49} = \frac{(0.42)(3.7^2)}{1.2 \sin 49}$$
$$F_T = 6.35N$$



7. You are at the carnival and decide to go on the swing ride. It is a high rotating platform from which the swing seats hang like pendulums. As the platform begins to turn, your swing's chain does not stay perpendicular to the ground, but angles out from the vertical. If your distance from the center of rotation is 8.50 m, and you go around once every 12.0 sec, what angle does your swing make with the vertical?



$$v = \frac{2\pi r}{T} = 4.45 m/s$$

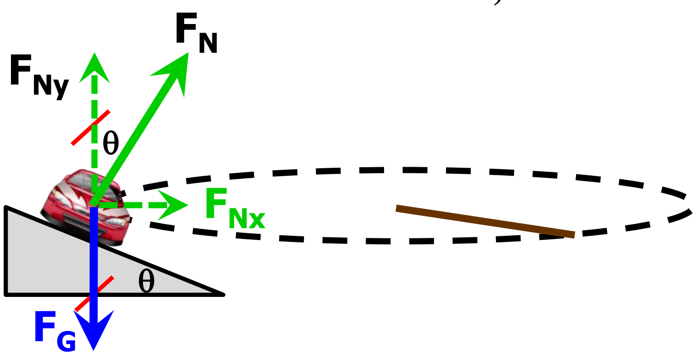
Newton's 2nd Law (y-dir)

$$\sum F_y = ma_y = 0$$
$$-F_g + F_{Ty} = 0$$
$$mg = F_{Ty} = F_T \cos\theta$$

Newton's 2nd Law (radial direction)

$$F_c = \sum F_r = ma_c$$
$$F_{Tx} = \frac{mv^2}{r}$$
$$F_T \sin\theta = \frac{mv^2}{r}$$
$$\frac{F_{Ty}}{\cos\theta} \sin\theta = \frac{mv^2}{r}$$
$$mg \tan\theta = \frac{mv^2}{r}$$
$$\theta = 13.4^o$$

8. Talladega Motor Speedway in Alabama has turns with radius 1,100 ft (335.3 m) that are banked at 33°. What is the "no friction" speed or design speed for a car on these turns (the speed for which no friction is required between the car's tires and the surface)?



$$F_c = \sum F_r = ma_c$$

$$F_{Nx} = \frac{mv^2}{r}$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$mg\tan\theta = \frac{mv^2}{r}$$

$$v = \sqrt{gr\tan\theta}$$

$$v = 46.2 m/s$$

$$\sum F_y = 0$$

$$F_{Ny} = mg$$

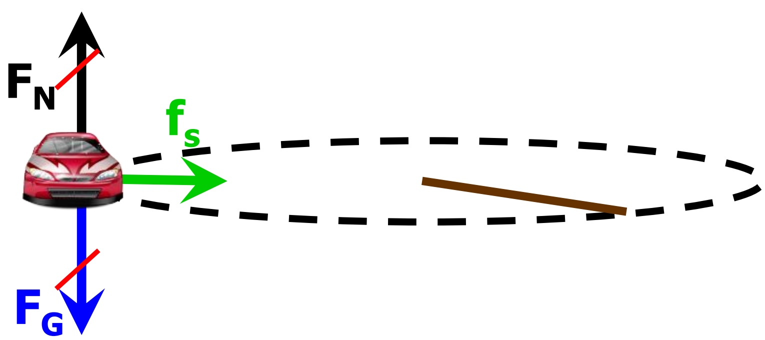
$$F_N \cos\theta = mg$$

$$F_N = \frac{mg}{\cos\theta}$$

9. A car drives around two circular curves on two different roads. The two curves have the same radius of curvature. Coincidentally, the maximum speed that the car can drive through either of the turns is the same for both roads. The same car drives through both turns at this same speed. The first road is frictionless, but is banked at 14^0 off the horizontal. The other turn is flat. What is the coefficient of friction between the car tires and the road on the unbanked turn?

In other words both cars have the same magnitude of centripetal force acting on them:

$Fc_{flat} = Fc_{bank}$

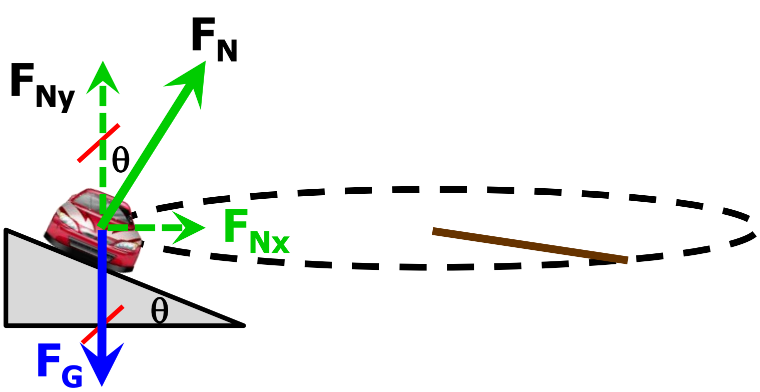


$$F_c = \sum F_r = ma_c$$

$$f_s = \frac{mv^2}{r}$$

$$\mu_s F_N = \mu_s mg = \frac{mv^2}{r}$$

$$F_{c\ flat} = \mu_s g = \frac{v^2}{r} \quad \textbf{(1)}$$



$$F_c = \sum F_r = ma_c$$

$$F_{Nx} = \frac{mv^2}{r}$$

$$\cos\theta = \frac{F_{Ny}}{F_N}$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$F_N = \frac{mg}{\cos\theta}$$

$$mg\tan\theta = \frac{mv^2}{r}$$

$$F_{cbank} = g\tan\theta = \frac{v^2}{r} \quad \textbf{(2)}$$

Combining (1) and (2):

$$F_{c\ flat} = F_{c\,bank}$$
$$\mu_s g = g\tan\theta$$
$$\mu_s = \tan\theta = \tan 14 = 0.25$$

10. A laboratory centrifuge operates at a rotational speed of 12,000 rpm (rev/min).
    a) What is the magnitude of the centripetal acceleration of a red blood cell at a radial distance of 8.00 cm from the centrifuge's axis of rotation?
    b) b) How does the acceleration compare with *g*?

f = 12000 rev/min = 200 rev/sec
T = 1/f = 0.005 sec

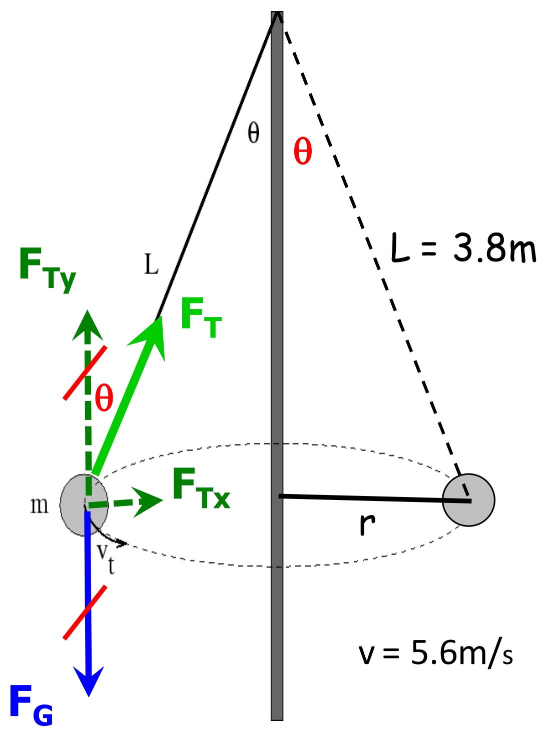
$$a_C = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(0.08)}{0.005^2} = 126{,}203 m/s^2 \approx 12{,}600 g's$$

11. (very challenging) A tetherball is suspended on a 3.8 m rope from a tall pole. The ball is hit so that it travels in a horizontal circle around the pole with a constant speed of 5.60 m/s. What angle does the rope make with the pole? (<u>Hint:</u> You cannot isolate θ, but you can set up a quadratic equation with cosθ as the unknown variable)

$$F_c = \sum F_r = ma_c$$
$$F_{Tx} = \frac{mv^2}{r}$$
$$mg\tan\theta = \frac{mv^2}{r}$$
$$g\tan\theta = \frac{v^2}{L\sin\theta}$$
$$\tan\theta\sin\theta = \frac{v^2}{Lg} = 0.842$$
$$\frac{\sin^2\theta}{\cos\theta} = \frac{1-\cos^2\theta}{\cos\theta} = 0.842$$
$$\cos^2\theta + 0.842\cos\theta - 1 = 0$$

$$\tan\theta = \frac{F_{Tx}}{F_{Ty}} = \frac{F_{Tx}}{mg}$$
$$F_{Tx} = mg\tan\theta$$

$$\sin\theta = \frac{r}{L}$$
$$r = L\sin\theta$$



This is a quadratic equation with *cosθ* as the variable. The roots are *cosθ = 0.664 and -1.506.*
Since magnitude of cosθ cannot be greater than 1 and for an angle less than 90°, the cosθ cannot be negative, we choose the solution
cosθ = 0.664. Then

$$\theta = \cos{-1}(0.664) = 48.4^o$$