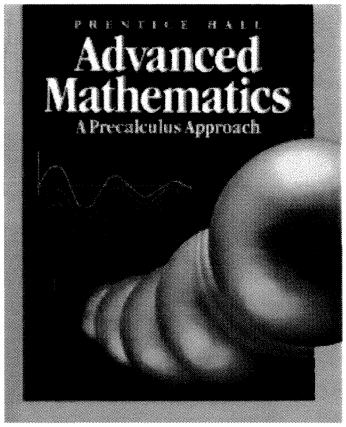
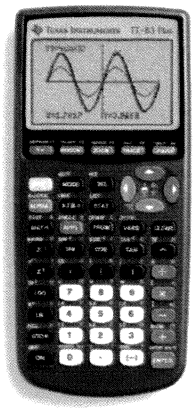


Pre-Calc Honors & Intro to Calculus BC Honors Midterm Exam REVIEW February, 2014



Name: _____ **Date:** _____

Teacher: _____ **Period:** _____

Your 140-point midterm examination will consist of:

- 55 multiple-choice questions worth a total of 110 points – these will be completed on the Scantron.
- 5 free response questions worth a total of 30 points (some with multiple parts) – these will be completed in the test booklet. You will need to show your work.

The exam will cover the following topics: Functions Review, Limits, Matrices, Sequences & Series, and (part of) Trigonometry.

You may use your calculator throughout the entire exam. Before the examination, clear your calculator of any formulas, notes or any such items, which could be perceived as "useful" or providing unfair advantage. The best solution is to RESET and clear the memory completely. The memory will be checked prior to the exam. If you forget your calculator on the day of the exam, your teacher will not provide you with a replacement for the exam.

NOTE: School policy mandates a penalty for cheating on an exam to be a grade of ZERO for that exam. The term cheating includes "intent to cheat." NO CELL PHONES. All cell phones must be kept out of sight. If a cell phone is seen during an exam, you will receive a grade of ZERO.

All calculators may be checked for inclusion of extraneous material. No papers should be placed in calculators. No information should be written on the front/back of calculators. The program portion of the graphing calculator will be checked. Any information entered there can be considered intent of cheating.

The following pages provide a review of the material which should be studied for this exam. Understand that just because a problem is not represented in this review package does NOT mean it cannot be asked on the exam. Please study your notes, homework, warm-ups, and assessments as well as this packet.

We will take some class time to review for this exam. Please feel free to stop in on your own time for further assistance. Good Luck!

Mr. Wieboldt & Ms. Eisen

PreCalc Honors & Intro to Calc BC Honors

Name_____

Midterm Exam Review 2014

Date_____ Period____

Expand completely.

1) $(y + 3x^3)^5$

Solve each equation.

2) $343^{-2a} \cdot 49^{-3a+1} = 343^{2a-3}$

3) $\left(\frac{1}{36}\right)^{-2r} \cdot 216 = 216^{r}$

4) $\ln(x-2) + \ln(x-12) = \ln 24$

Divide.

5) $(5b^5 - 31b^4 - 58b^3 - 59b^2 - 23b + 10) \div (5b + 4)$

Identify the domain and range of each.

6) $y = \log_2(2x+1) + 5$

7) $y = \frac{2}{3}\sqrt[3]{x+5} + 3$

8) $y = 1 + \frac{3}{4}\sqrt[3]{x-2}$

Solve each equation. Round your answers to the nearest ten-thousandth.

9) $4e^{-3k-6} - 5 = 25$

Find the inverse of each function.

10) $y = \left(\frac{e^x - 1}{-4}\right)^{\frac{1}{2}}$

11) $g(n) = -\frac{3}{n+2} - 2$

Condense each expression to a single logarithm.

12) $\log_6 x + \frac{\log_6 u}{3} + \frac{\log_6 v}{3} + \frac{\log_6 w}{3}$

Use the properties of logarithms and the logarithms provided to rewrite each logarithm in terms of the variables given.

13) $\log_5 8 = A$
$\log_5 6 = B$
$\log_5 11 = C$
Find $\log_5 \frac{22}{9}$

Evaluate each function at the given value.

14) $f(a) = a^4 + \frac{10}{3}a^3 - \frac{23}{3}a^2 + \frac{10}{3}a - \frac{10}{7}$ at $a = \frac{2}{3}$

Sketch the general shape of each function.

15) $f(x) = -x^2 + 8x - 15$

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.

16) $f(x) = \frac{x+3}{3x^2 - 3x - 36}$

17) Chelsea and Eugene each improved their yards by planting grass sod and shrubs. They bought their supplies from the same store. Chelsea spent \$132 on 7 ft² of grass sod and 9 shrubs. Eugene spent \$130 on 5 ft² of grass sod and 10 shrubs. What is the cost of one ft² of grass sod and the cost of one shrub?

18) Simplify.
(n!)^2

(n-1)! (n+1)!

19) Simplify.
$\frac{\cot^2 x}{1 + \csc x} + \sin x \cdot \csc x$

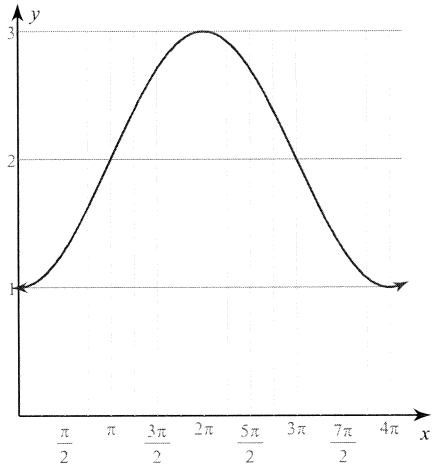
20) Prove the given identity.
$\tan^2 x - \sin^2 x = \tan^2 x \cdot \sin^2 x$

21) A unicycle has a tire with radius 10 in. It's traveling at a speed of 5.5 mph. Find the angular velocity of the tire in radians per second.

22) Evaluate.
sin(Arcsin(1/2))

23) Rewrite y=sec($\cot^{-1} x$) as algebraic equation.

24) Determine the equation for the graph.



25) Fully decompose into partial fractions.

$$\frac{3x^2 + 3x + 7}{(x+3)^2(x-2)}$$

Find the missing term or terms in each arithmetic sequence.

26) ..., 16, ___, ___, ___, ___, 166, ...

27) ..., 16, ___, ___, ___, ___, 26, ...

Given two terms in an arithmetic sequence find the term named in the problem, the explicit formula, and the recursive formula.

28) $a_{14} = 81$ and $a_{38} = 201$
Find a_{33}

29) $a_{10} = -11$ and $a_{37} = -47$
Find a_{30}

Determine the number of terms *n* in each geometric series.

30) $\sum_{i=1}^{n} -2 \cdot 4^{i-1} = -10922$

Determine the number of terms *n* in each arithmetic series.

31) $\sum_{k=1}^{n} (1.4k + 3.7) = 51.6$

Rewrite each series using sigma notation.

32) $605 + 606 + 607 + 608 + 609 + 610$

33) $1 + 5 + 25 + 125 + 625 + 3125$

Given two terms in a geometric sequence find the term named in the problem, the explicit formula, and the recursive formula.

34) $a_4 = 50$ and $a_3 = -10$
Find a_9

35) $a_6 = -729$ and $a_3 = -27$
Find a_{12}

Find the missing term or terms in each geometric sequence.

36) ..., $-\frac{125}{16}$, ___, ___, ___, ___, $-\frac{64}{25}$, ...

Determine if each geometric series converges or diverges.

37) $-3 + 6 - 12 + 24...$

Evaluate each infinite geometric series described.

38) $\sum_{m=1}^{\infty} 3 \cdot \left(\frac{2}{3}\right)^{m-1}$

39) $-0.7 - 0.63 - 0.567 - 0.5103...$

Determine the common ratio of the infinite geometric series.

40) $a_1 = 8,\ S = 16$

Simplify. Write "undefined" for expressions that are undefined.

41) $\left(\begin{bmatrix} 3 & -6 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ 1 & 1 \end{bmatrix}\right) \cdot \begin{bmatrix} -5 & 2 \\ -6 & 6 \end{bmatrix}$

42) $\left(\begin{bmatrix} -2 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -6 \end{bmatrix}\right) \cdot \begin{bmatrix} -5 & -4 \\ -3 & 5 \\ -1 & 5 \end{bmatrix}$

43) $\begin{bmatrix} -4 & -1 & 1 \\ -1 & -1 & 4 \end{bmatrix} + \begin{bmatrix} 6 & -3 & -1 \\ -1 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 4 & -3 & 2 & 2 \end{bmatrix}$

Evaluate each determinant.

44) $\begin{vmatrix} -6 & 4 \\ -6 & -8 \end{vmatrix}$

45) $\begin{vmatrix} -3 & -2 & 3 \\ -6 & -7 & -7 \\ -6 & 7 & -5 \end{vmatrix}$

For each matrix state if an inverse exists.

46) $\begin{bmatrix} 0 & -6 \\ 0 & -10 \end{bmatrix}$

47) $\begin{bmatrix} 4 & 3 & 0 \\ 2 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

Find the inverse of each matrix.

48) $\begin{bmatrix} -2 & 5 \\ 6 & -1 \end{bmatrix}$

49) $\begin{bmatrix} 3 & -6 & -4 \\ -1 & -5 & 1 \\ -5 & 4 & -4 \end{bmatrix}$

Solve each equation or state if there is no unique solution.

50) $\begin{bmatrix} -8 & -2 & 3 \\ -7 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 0 & -2 \end{bmatrix} X = \begin{bmatrix} -22 & 8 & -11 \\ -21 & 11 & -19 \end{bmatrix}$

51) $\begin{bmatrix} 4 & -9 \\ 1 & -2 \end{bmatrix} Y - \begin{bmatrix} 7 & 1 \\ -8 & 1 \end{bmatrix} = \begin{bmatrix} -30 & 18 \\ 3 & 4 \end{bmatrix}$

Use Cramer's Rule to solve each system.

52) $-3y + z = -12$
$-6x - 6y - 2z = -24$
$3x - 5y - 2z = -20$

53) $2x - 2y = 12$
$x + 6y - 5z = 5$
$3x - y - 5z = -3$

Convert each degree measure into radians and each radian measure into degrees.

54) -150°

55) $-\frac{25\pi}{9}$

Find a positive and a negative coterminal angle for each given angle.

56) $\frac{31\pi}{10}$

57) $\frac{71\pi}{36}$

Find the length of each arc.

58) $r = 8$ km, $\theta = \frac{\pi}{3}$

Find the area of each sector.

59) $r = 5$ yd, $\theta = \frac{7\pi}{4}$

Find the reference angle.

60) $\frac{19\pi}{6}$

61) $-\frac{2\pi}{3}$

Solve each equation for $0 \le \theta < 360$.

62) $-5 - 4\tan\theta = -5$

63) $4 = 4 - \tan\theta$

Find the value of the trig function indicated.

64) Find $\tan\theta$ if $\cot\theta = \frac{24}{7}$

65) Find $\sec\theta$ if $\cos\theta = \frac{\sqrt{2}}{2}$

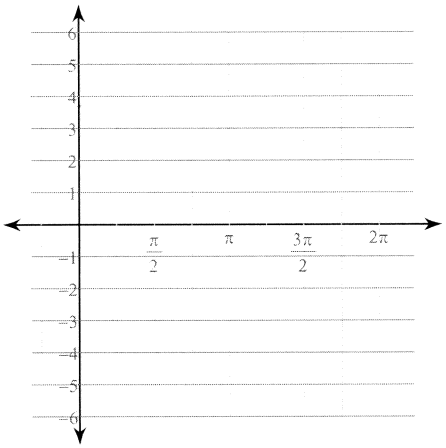
Use the given point on the terminal side of angle θ to find the value of the trigonometric function indicated.

66) $\cos\theta; (-\sqrt{7}, 3)$

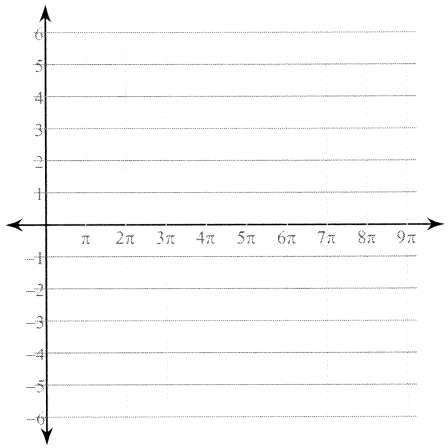
67) $\cos\theta; (2\sqrt{3}, 2)$

Using radians, find the amplitude and period of each function. Then graph.

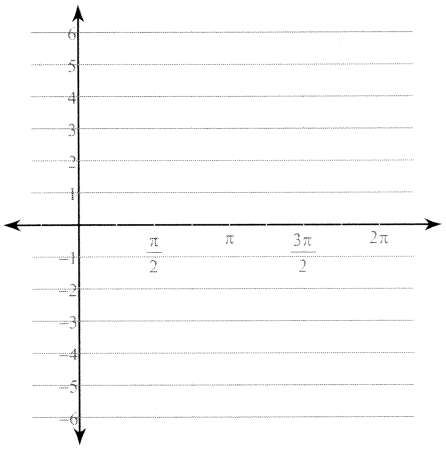
68) $y = 2 + \frac{1}{2} \cdot \sin\left(3\theta - \frac{\pi}{6}\right)$



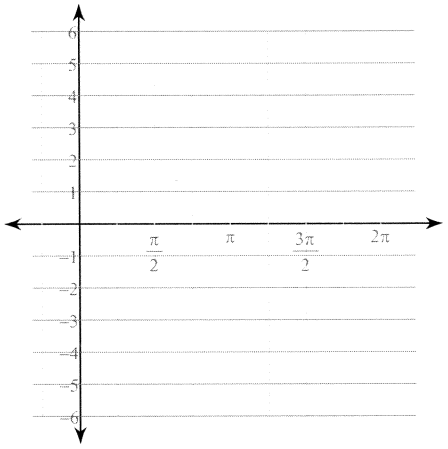
69) $y = 3\sin\left(\frac{\theta}{3} + \frac{3\pi}{4}\right) - 1$



70) $y = \csc\left(-\frac{\theta}{2} + \frac{\pi}{2}\right)$

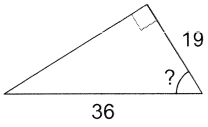


71) $y = 2\csc\left(2\theta + \frac{\pi}{4}\right) - 2$



Find the measure of the indicated angle to the nearest degree.

72)



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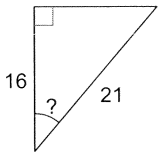

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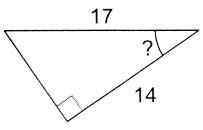

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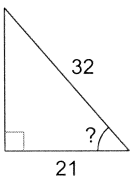


Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

76) $f(x) = -2\tan(x);\ [-\pi, \pi]$

77) $f(x) = \begin{cases} -\frac{x}{2} - \frac{9}{2}, & x \le 1 \\ -x^2 - 2x - 2, & x > 1 \end{cases}$

78) $f(x) = \begin{cases} 2, & x < -2 \\ -x + 2, & x \ge -2 \end{cases}$

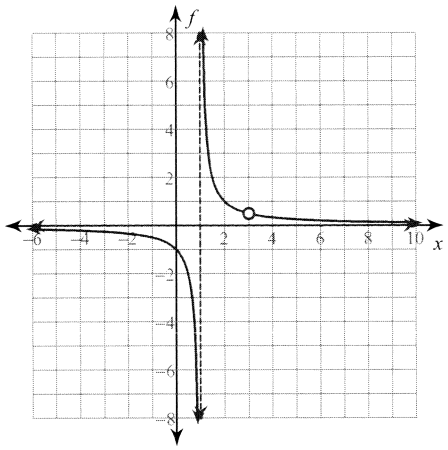
79) $f(x) = \dfrac{x + 6}{x^2 + x - 6}$

80) $f(x) = \sin\dfrac{1}{x}$

81) $f(x) = \begin{cases} x^2 + 8x + 17, & x \le -3 \\ -\frac{x}{2} + 4, & x > -3 \end{cases}$

Determine if each function is continuous. If the function is not continuous, find the *x*-axis location of and classify each discontinuity.

82) $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$



Evaluate each limit.

83) $\lim_{x\to 0^+} \csc(2x)$

84) $\lim_{x\to \frac{2\pi}{3}^-} 2\sec(x)$

85) $\lim_{x\to 3^+} \dfrac{x-3}{x^2-2x-3}$

86) $\lim_{x\to -\infty} \sqrt{\dfrac{3x^2+5}{2x^2+1}}$

87) $\lim_{x\to -\infty} \dfrac{\sqrt{x^2+3}}{2x-3}$

88) $\lim_{x\to \infty} \dfrac{x-2}{\sqrt{2x^2+3}}$

89) $\lim_{x\to 3} \dfrac{3|x-3|}{x-3}$

90) $\lim_{x\to -3} f(x),\ f(x) = \begin{cases} x^2+6x+9, & x \le -3 \\ -\dfrac{x}{2} - \dfrac{3}{2}, & x > -3 \end{cases}$

91) $\lim_{x\to 3} \dfrac{-3x+9}{|-x+3|}$

92) $\lim_{x\to 3} -\sqrt[3]{2x-3}$

93) $\lim_{x\to 1} -\sqrt[3]{-x-4}$

94) $\lim_{x\to -2} \sqrt[3]{x-4}$

95) $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x+3}-2}$

96) $\lim_{x\to 1} \dfrac{\sqrt{x+15}-4}{x-1}$

Answers to Midterm Exam Review 2014

1) $y^5 + 15y^4x^3 + 90y^3x^6 + 270y^2x^9 + 405yx^{12} + 243x^{15}$

2) $\left\{\frac{11}{18}\right\}$

3) $\{-3\}$

4) $\{14\}$

5) $b^4 - 7b^3 - 6b^2 - 7b + 1 + \frac{6}{5b+4}$

6) Domain: $x > -\frac{1}{2}$
Range: All reals

7) Domain: { All real numbers. }
Range: { All real numbers. }

8) Domain: { All real numbers. }
Range: { All real numbers. }

9) –2.6716

10) $y = \ln\left(-4x^2 + 1\right)$

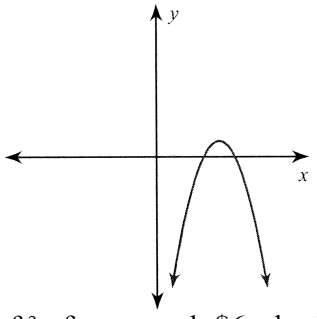
11) $g^{-1}(n) = -\frac{3}{n+2} - 2$

12) $\log_6\left(x\sqrt[3]{wvu}\right)$

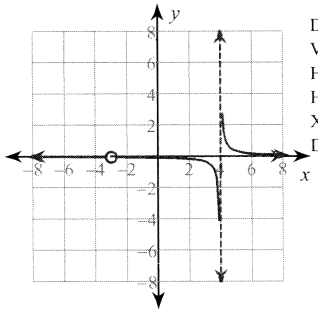
13) $A + C - 2B$

14) $-\frac{10}{7}$

15)



16)



Discontinuities: 4, –3
Vertical Asym.: $x = 4$
Holes: $x = -3$
Horz. Asym.: $y = 0$
X-intercepts: None
Domain:
All reals except 4, –3

17) ft² of grass sod: \$6, shrub: \$10

18) $\frac{n}{n+1}$

19) csc x

20) Done in class.

21) 9.8 rad/s

22) 1/2

23) $y = \frac{\sqrt{x^2+1}}{x}$

24) $y = \sin\left(0.5(x - \pi)\right) + 2$

25) $\frac{2}{x+3} - \frac{5}{(x+3)^2} + \frac{1}{x-2}$

26) 46, 76, 106, 136

27) 18, 20, 22, 24

28) $a_{33} = 176$
Explicit: $a_n = 11 + 5n$
Recursive: $a_n = a_{n-1} + 5$
$a_1 = 16$

29) $a_{30} = -\frac{113}{3}$
Explicit: $a_n = \frac{7}{3} - \frac{4}{3}n$
Recursive: $a_n = a_{n-1} - \frac{4}{3}$
$a_1 = 1$

30) 7

31) 6

32) $\sum_{k=5}^{10}(k + 600)$

33) $\sum_{k=0}^{5} 5^k$

34) $a_9 = -156250$

Explicit: $a_n = -0.4 \cdot (-5)^{n-1}$

Recursive: $a_n = a_{n-1} \cdot -5$

$a_1 = -0.4$

35) $a_{12} = -531441$

Explicit: $a_n = -3 \cdot 3^{n-1}$

Recursive: $a_n = a_{n-1} \cdot 3$

$a_1 = -3$

36) $-\frac{25}{4}, -5, -4, -\frac{16}{5}$

37) Diverges

38) 9

39) -7

40) $\frac{1}{2}$

41) $\begin{bmatrix} 36 & -54 \\ -62 & 50 \end{bmatrix}$

42) $\begin{bmatrix} 3 & -25 \end{bmatrix}$

43) Undefined

44) 72

45) -528

46) No

47) No

48) $\begin{bmatrix} \frac{1}{28} & \frac{5}{28} \\ \frac{3}{14} & \frac{1}{14} \end{bmatrix}$

49) $\frac{1}{218} \cdot \begin{bmatrix} 16 & -40 & -26 \\ -9 & -32 & 1 \\ -29 & 18 & -21 \end{bmatrix}$

50) No unique solution

51) $\begin{bmatrix} 1 & 7 \\ 3 & 1 \end{bmatrix}$

52) $(0, 4, 0)$

53) $\left(10, 4, \frac{29}{5}\right)$

54) $-\frac{5\pi}{6}$

55) -500°

56) $\frac{11\pi}{10}$ and $-\frac{9\pi}{10}$

57) $\frac{143\pi}{36}$ and $-\frac{\pi}{36}$

58) $\frac{8\pi}{3}$ km

59) $\frac{175\pi}{8}$ yd²

60) $\frac{\pi}{6}$

61) $\frac{\pi}{3}$

62) $\{0, 180\}$

63) $\{0, 180\}$

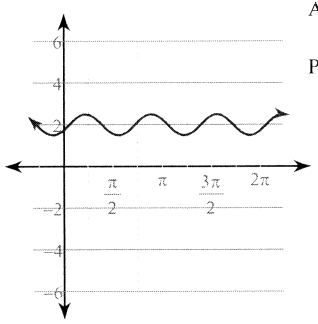
64) $\frac{7}{24}$

65) $\sqrt{2}$

66) $-\frac{\sqrt{7}}{4}$

67) $\frac{\sqrt{3}}{2}$

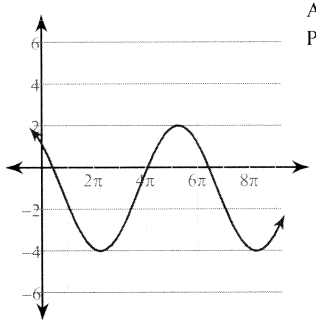
68)



Amplitude: $\frac{1}{2}$

Period: $\frac{2\pi}{3}$

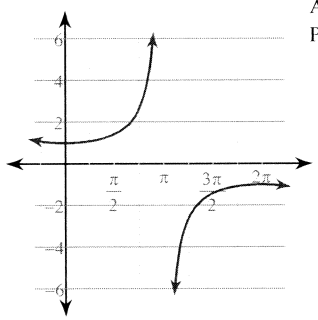
69)



Amplitude: 3

Period: 6π

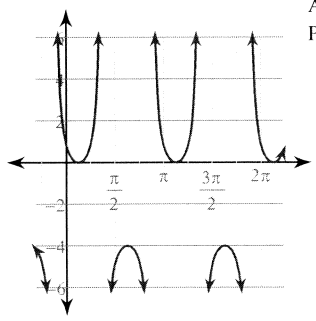
70)



Amplitude: None

Period: 4π

71)



Amplitude: None

Period: π

72) 58°

73) 40°

74) 35°

75) 49°

76) Essential discontinuities at: $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

77) Continuous

78) Jump discontinuity at: $x = -2$

79) Essential discontinuities at: $x = -3, x = 2$

80) Oscillating discontinuity at: $x = 0$

81) Jump discontinuity at: $x = -3$

82) Point discontinuity at: $x = 3$
Essential discontinuity at: $x = 1$

83) Does not exist.

84) -4

85) $\frac{1}{4}$

86) $\frac{\sqrt{6}}{2}$

87) $-\frac{\sqrt{1}}{2}$

88) $\frac{1}{\sqrt{2}}$

89) Does not exist.

90) 0

91) Does not exist.

92) $-\sqrt[3]{3}$

93) $\sqrt[3]{5}$

94) $-\sqrt[3]{6}$

95) 4

96) $\frac{1}{8}$