Radial Net Forces and Circular Motion Problems

For each of the problems below, begin your solution with a clear, accurate free body diagram. Show your solutions step by step starting with the basic conceptual equation (Newton’s 2nd Law). Use separate pieces of paper to solve these problems.

1. A woman flying aerobatics executes a maneuver as illustrated below:

a. Construct a force diagram of all relevant forces acting on the woman flying the airplane at the TOP of the loop (when upside-down) and at the BOTTOM of the loop (right side up).

b. Determine the normal force on the woman at the top of the loop.

**Newtons 2nd Law (radial)**

\[ F_c = \sum F_r = ma_c \]

\[ F_g + F_N = \frac{mv^2}{r} \]

A centripetal force of 616.8N (mv²/r) is necessary for the pilot to fly in a circular loop at the given mass, radius and speed. At the top of the loop, the 616.8N centripetal force is supplied by a combination of gravity and the normal force \((F_g + F_N)\). Gravity contributes 539N of the centripetal force and the normal force contributes 77.8N.

\[ F_N = \frac{mv^2}{r} - mg \]

\[ = 616.75 - 539 \]

\[ F_N = 77.8N \]

c. Determine the normal force on the woman at the bottom of the loop. How many G’s of force does the pilot experience at the bottom of the loop? (One G force on the pilot is the force of her weight. \#G=F_N/F_g)

**Newtons 2nd Law (radial)**

\[ F_c = \sum F_r = ma_c \]

\[ -F_g + F_N = \frac{mv^2}{r} \]

\[ F_N = \frac{mv^2}{r} + mg \]

\[ = 616.75 + 539 \]

\[ F_N = 1155.75N = 1155.75N \times \frac{1G}{mg} = 2.14G's \]

A centripetal force of 616.8N (mv²/r) is necessary for the pilot to fly in a circular loop at the given mass, radius and speed. At the bottom of the loop, the 616.8N centripetal force is supplied by a combination of gravity and the normal force \((-F_g + F_N)\). The normal force contributes 1155.8N of centripetal force and gravity reduces the centripetal force by 539N.

d. What speed does the plane have to go for the pilot to feel weightless at the top?

To feel weightless at the top, the only force acting on the pilot should be \(F_g\) (free fall) and \(F_N = 0\). Therefore, the centripetal force at the top would be supplied only by gravity \((F_c = F_g)\) and the centripetal acceleration at the top should be that due to gravity \((a_c = g)\).
\[ F_c = \sum F_r = ma_c \]
\[ F_g + 0 = \frac{mv^2}{r} \]
\[ mg = \frac{mv^2}{r} \quad g = \frac{v^2}{r} \]
\[ v = \sqrt{gr} = 61m/s \]

e. At the speed in part d, what would the pilot’s apparent weight be at the bottom? What would be the G force experienced at the bottom?

\[ F_c = \sum F_r = ma_c \]
\[-F_g + F_N = F_g \]
\[ F_N = 2F_g = 2G's = 1078N \]

We saw that at the speed for the pilot to be weightless, the centripetal force at the top is \( F_g \). Since the magnitude of \( F_c \) is the same at every point on the circle \((mv^2/r)\), then the magnitude of \( F_c \) at the bottom is also \( F_g \).

2. Assume that the car shown below is going at a constant speed

a. Construct a force diagram for the car when it’s at the top of the hill.

Note that at the top of the hill, the centripetal force must be directly down, towards the center of the circle. Therefore, \( F_N < F_g \) in order to get a downwards net force.

b. Suppose the speed of the car is 11.1 m/s \((\approx 25\ mph)\), its mass is 1200 kg and the radius of curvature \((r)\) is 25 m; determine the normal force acting on the car.

Centripetal force is supplied by combination of gravity (pointing into circle) and the normal force (pointing out of circle) \((F_c=F_g-F_N)\). It is \(5914.1\ N\) \((mv^2/r)\).

The normal force could represent what a scale would read of your weight. Here the normal force is your true weight minus a term due to the centripetal acceleration. Your apparent weight is lighter as you go over the top of the hill due to the acceleration, of the car.

c. How fast would the car be traveling if the normal force were reduced to zero? This occurs the instant the car loses contact with the hill.

If the car went faster and faster over the top, the centripetal force needed to make the curve would have to increase. To do this, the normal force would shrink. At some speed, the normal force goes to zero and the only force providing the centripetal force would be gravity and the car would feel weightless at the top.
\[
\sum F_r = ma_c \\
F_g = \frac{mv^2}{r} \\
mg = \frac{mv^2}{r} \\
g = \frac{v^2}{r} \quad (g = a_c) \\
v = \sqrt{gr} = 15.7m/s \quad (35mph)
\]

d. Suppose the car were going faster than the speed that you calculated for part c; describe what would happen to the car.
There would no longer be enough centripetal force to hold the car on the road. The car would fly off the top of the hill horizontally, with a horizontal velocity. It would be then act like horizontally launched projectile.

3. A 0.25 kg yo-yo is being swung in a vertical circle at the end of a string that is 0.70m long at a constant speed of 7.50m/s.
   a) What is the centripetal acceleration of the yo-yo? Same \( m, v \) and \( r \) at every position of the circular path, so \( F_c \) is the same magnitude at every position.
      \[ a_c = \frac{v^2}{r} = \frac{7.5^2}{0.7} = 80.4m/s^2 \]
   b) What is the centripetal force on the yo-yo?
      \[ F_c = ma_c = 0.25(80.4) = 20.1N \]
   c) Construct a FBD of the yo-yo at the bottom of the loop, the top of the loop and the side of the loop and explain where the tension of the string is maximum.
      The tension of the string is maximum at the bottom because at the bottom the tension has to provide all of the centripetal force plus the full weight of the ball since \( mg \) is pointing out of the circle and therefore reducing the centripetal force.
      Tension is minimum at the top where all of the force of gravity is contributing to the centripetal force and so the string doesn’t have to pull as hard.

d) What is the tension in the string at the bottom of the circle? At the top of the circle? At the side of the circle?

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e) If the string can withstand 50 N of force before breaking, what is the maximum speed the yo-yo could move before the string breaks? (Hint: look at the position where the tension is maximum in the circle)

Because the tension is maximum at the bottom, it will reach max at the bottom when the speed is maximum At Bottom

\[ F_c = \sum F_r = ma_c = \frac{mv_{\text{max}}^2}{r} \]

\[ F_{T\,\text{max}} - F_g = \frac{mv_{\text{max}}^2}{r} \]

\[ v_{\text{max}} = \sqrt{r(F_{T\,\text{max}} - F_g)/m} \]

\[ v_{\text{max}} = \sqrt{0.7(50 - 2.45)/0.25} = 11.5m/s \]

f) Determine the minimum speed the yo-yo could go around at the given radius. What would happen if the yo-yo travelled less than the minimum speed? At the top is where minimum speed is an issue because as the speed decreases, less centripetal force is required for the circular motion and so the tension decreases until it reaches zero. The speed at which \( F_T = 0 \) is the minimum speed because it is not possible to decrease the speed more and stay in the 0.7m radius circular path because \( mg \) is the only centripetal force and it cannot be reduced. Therefore, at less than \( v_{\text{min}} \), the yo yo will fall out of the circle because it doesn’t have the necessary tangential speed to hold the circular path.

\( V_{\text{min}} \) is at the top when \( F_T = 0 \) and \( F_g \) is the only centripetal force.

\[ F_c = \sum F_r = ma_c = \frac{mv_{\text{min}}^2}{r} \]

\[ F_c = F_g = mg = \frac{mv_{\text{min}}^2}{r} \]

\[ g = \frac{v_{\text{min}}^2}{r} \quad \text{(at the top, } a_c = g, \text{ yo-yo is momentarily in free fall)} \]

\[ v_{\text{min}} = \sqrt{gr} = 2.6m/s \]

Notice that this is the expression for the speed of an object in UCM when the force of gravity is the ONLY centripetal force.

4. The amusement park ride shown in the diagram operates as follows: riders enter the cylindrical structure when it is stationary with the floor at the point marked "a". They then stand against the wall as the cylinder then begins to rotate. When it is up to speed, the floor is lowered to the position marked "b", leaving the riders "suspended" against the wall high above the floor. This starts when the normal force is equal to half of the rider’s weight (regardless of how heavy or light they are, trust me!)

a. Draw a force diagram for the rider

\[ r = 1.5 \text{ m} \]
\[ m_b = 73 \text{ kg} \]
b. Determine the speed when the floor begins to lower. The floor lowers when $F_N = \frac{1}{2} F_g$

Centripetal force is supplied by the normal force of the wall of the gravitron.

The floor drops when $F_N = \frac{1}{2} F_g$.

Notice that the speed at which the floor drops does NOT depend on the mass of the passengers. It only depends on the radius of the Gravitron and the acceleration due to gravity.

$$v = \sqrt{\frac{gr}{2}} = 2.71\text{ m/s} \quad (6.1\text{ mph})$$

c. Determine the period of rotation when the floor begins to lower.

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi (1.5)}{2.71} = 3.48s$$

Notice at the speed at which the floor lowers, the full weight of the rider stuck on the wall must be balanced by static friction. That is A LOT of friction – imagine trying to “hang” on a wall stuck there only by friction. The friction acts like glue. The friction is much larger than normal because as the gravitron spins, the wall is continually being pressed into the rider and that makes the normal force grow large. The rider experiences that pressing force as an outwardly pressing “centrifugal force”. When the floor drops and $F_N = \frac{1}{2} F_g$

$$\sum F_y = ma_y = 0$$

$$f_s - F_g = 0$$

$$f_s = F_g$$

$$\mu_s F_N = F_g$$

$$\mu_s (1/2 F_g) = F_g$$

$$\mu_s = 2$$

5. Amusement park rollercoaster rides use a hill for riders to gain speed followed by an upside down loop. The loops are designed with large radius bottoms and small radius tops, and such a shape is called a clothoid. Answer the following questions in order to find out why the clothoid is used.

The speed of the rollercoaster is 22 m/s at the bottom of the loop and 8 m/s at the top of the loop.
Draw force diagrams and determine the size of the force on the rider by the seat at the top and bottom of the loop. Assume the rider has a mass of 55 kg.

Note that at the top, the centripetal force must be directly down, towards the center of the circle. Both $F_N$ and $F_g$ are contributing to the centripetal force at the top.

Note that at the bottom, the centripetal force must be directly up, towards the center of the circle. Therefore, $F_N > F_g$ in order to get a upwards net force.

**BOTTOM**

Centripetal force is supplied by combination of gravity and the normal force ($F_N-F_g$). It is $2662\, N$ ($mv^2/r$)

The normal force represents apparent weight. At the bottom, the normal force is the true weight plus a term due to the centripetal acceleration. The apparent weight is heavier as the rider goes through the bottom of the hill due to the acceleration.

You are subjected to $5.9G's$ of force.

**TOP**

Centripetal force is supplied by combination of gravity and the normal force ($F_N+F_g$). It is $704\, N$ ($mv^2/r$)

The normal force represents apparent weight. At the top, the normal force is less than the true weight of the rider. The apparent weight is lighter as the rider goes past the top of the loop due to the acceleration of the coaster.
6. Suppose the speed of the car in figure is 15.6 m/s (≈ 35 mph), its mass is 1200 kg and the radius of curvature (r) is 23 m.

At the bottom, the centripetal force must be directly up, towards the center of the circle. Therefore, \( F_N > F_g \) in order to get a upwards net force.

a. Determine the normal force acting on the car.

Centripetal force is supplied by combination of gravity and the normal force \((F_N - F_g)\). It is \(12,697 \text{ N} \) \(\left( \frac{mv^2}{r} \right)\)

The normal force represents apparent weight. Here the normal force is more than the true weight of the car by an amount due to the acceleration of the car. The normal force is about double the weight of the car so it subjects the car to a force of about 2G’s.

b. How fast does the car need to be moving in order for it to “feel” a normal force equal to three times its weight at the bottom of the dip. This is also known as experiencing 3 G’s of force.

7. 80 kg Tarzan grabs a vine to swing to another tree.

As Tarzan sings from point A to point B, describe qualitatively how the tension in the vine changes and why. As the rope swings from A to B, the component of gravity along the radial direction increases until at B, all of gravity is in the radial direction and it is pointing out of the circle. Since the radial component of gravity points away from the center of the circle, increasingly counteracts the centripetal force so the tension in string must grow to maintain the centripetal force.

b) At point B he is swinging at 7 m/s and the vine is 8 meters long. How hard does he have to hold on to the vine to keep from slipping off?
If the rope tension on Tarzan is 1274 N, then Tarzan must hold on with an equal and opposite force.

\[ F_c = \sum F_r = ma_c \]

\[ F_T - F_g = \frac{mv^2}{r} \]

\[ F_T = mg + \frac{mv^2}{r} \]

\[ = 784 + 490 \]

\[ F_T = 1274N \]

c) A moment later, at point C, the vine catches on a branch, reducing the radius of the swing to 2 m. If Tarzan is still travelling at 7 m/s, how hard does he now have to hold on to the vine?

\[ \sum F_r = ma_c \]

\[ F_T - F_g = \frac{mv^2}{r} \]

\[ F_T = mg + \frac{mv^2}{r} \]

\[ = 784 + 1260 \]

\[ F_T = 2044N \]

d) If Tarzan slips off at point C what will be the path he takes? (Sketch the path on the diagram.) How does this path differ from circular motion?

If he slipped off at C, he would have a launch velocity tangent to the path he was following at point C. Once airborne, he would follow the parabolic trajectory of projectile motion (if we neglect the effect of air resistance).