Name_______________________________ **Date**______
AP Physics 1

Simple Harmonic Motion and Springs

Hookean Spring

$$F_S = -k\Delta x$$

$$-W_S = U_S = -\tfrac{1}{2}k(\Delta x^2)$$

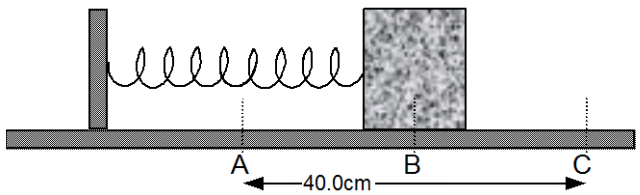
Simple Harmonic Motion of Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$x = A\cos(\omega t) = A\cos(2\pi f t)$$

$$a = \frac{F}{m} = \frac{-kx}{m}$$

1. What are the two criteria for simple harmonic motion?
   - Only restoring forces cause simple harmonic motion. A restoring force is a force that it proportional to the displacement from equilibrium and in the opposite direction.
   - Position, velocity and the other variables of simple harmonic motion are sinusoidal functions of time.
2. The diagram to the right shows a 2 kg block attached to a Hookean spring on a frictionless surface. The block experiences no net force when it is at position B. When the block is to the left of point B the spring pushes it to the right. When the block is to the right of point B, the spring pulls it to the left.



The mass is pulled to the right from point B to point C and released at time t = 0. The block then oscillates between positions A and C. Assume that the system consists of the block and the spring and that no dissipative forces act.

a) The block takes 40.0 s to make 20 oscillations. What is the "period of oscillation" for this system?

T = 40s/20 oscillations = 2 sec/oscillation

b) What is the frequency of this oscillating system?

f = 1/T = ½ = 0.5 Hz

c) What is the amplitude of vibration of this system?

A = 0.2 m

d) Determine the spring constant of the spring.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$2 = 2\pi\sqrt{\frac{2}{k}}$$

$$k = 19.7 N/m$$

e) Write an equation that describes the position of the mass as a function of time, starting from position C at time t =0.

$$x = 0.2\cos(2\pi f t) = 0.2\cos(\pi t)$$

f) Explain what would happen to the period and frequency of this system if you were to double the amplitude while keeping the mass and spring constant the same. The period and frequency would not change since they do not depend on the amplitude of oscillation

g) Explain what would happen to the period and frequency of this system if you were to double the mass while keeping the amplitude and spring constant the same.
period depends on square root of mass so if mass doubled, T would change by √2
frequency depends on 1/(√m) so if mass doubled, f would change by 1/√2 = 0.71

$$T = 2\pi\sqrt{m/k} \quad 2xm \Rightarrow \sqrt{2}xT = 1.41T$$

$$1/\sqrt{2}\, f = 0.71f$$

h) Explain what would happen to the period and frequency of this system if you were to double the spring constant while keeping the amplitude and mass constant.
period depends on 1/(√k) so if spring constant doubled, T would change by 1/√2 = 0.71x
frequency depends on square root of k so if spring constant doubled, f would change by √2x

$$T = 2\pi\sqrt{m/k} \quad 2xk \Rightarrow 1/\sqrt{2}\, xT = 0.71T,$$

$$\sqrt{2}f = 1.41f$$

i) Determine the amount of energy of the oscillating spring and mass system.
The energy of the block and spring is conserved since only the spring force (a conservative force) does work. Therefore $E_A = E_B = E_C$

$$E = E_A = ½\, k(\Delta x^2) = ½\,(19.7)(0.2^2) = 0.394J$$

j) Where does the block have maximum speed and what is the maximum speed?
Block has maximum speed at point B, the equilibrium point of the spring. At that point, all the energy is in the form of kinetic energy

$$E = E_B = \tfrac{1}{2}mv_B^2$$

$$0.394 = \tfrac{1}{2}(2)v_B^2$$

$$V_B = 0.63m/s$$

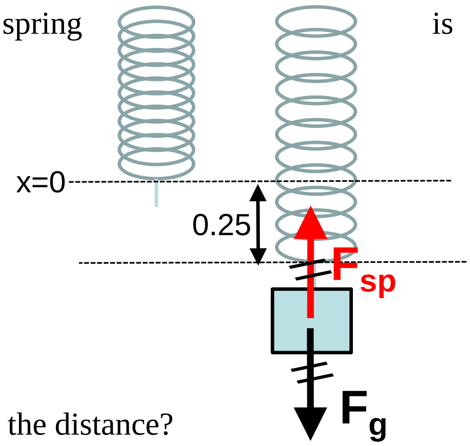
k) Where does the block experience maximum acceleration and what is the maximum acceleration?
Block has maximum acceleration at points A and C, at max displacement from equilibrium, where $F_{net} = Fs_{pring}$ is greatest

$$a_C = \frac{F_S}{m} = \frac{-kx_C}{m} = \frac{-(19.7)(0.2)}{2} = 1.97m/s^2$$

l) What would happen to the energy if the amplitude of oscillation were tripled while keeping the mass and spring constant unchanged?
The energy is proportional to the amplitude squared ($E_A = ½\, kA^2$). Therefore, if the amplitude of the oscillation were tripled, the total energy would increase 9xs.

m) What would happen to the maximum speed if the amplitude of oscillation were tripled while keeping the mass and spring constant unchanged?
The max speed at equilib is proportional to the amplitude ($E_A = E_B = ½\, kA^2 = ½\, mv_B^2$). Therefore, if the amplitude of the oscillation were tripled, the max speed would also triple.

3. A 5 kg mass is attached to a spring that is hanging vertically. The spring is stretched 0.25 m from its equilibrium position.



a) What is the spring constant?

$$F_S = F_g$$
$$k\Delta x = mg$$
$$k = \frac{mg}{\Delta x} = 196 N/m$$

b) What mass would be required to stretch the spring three times the distance?

3xs or 15 kg since the mass is directly proportional to the displacement

The 5 kg object hanging on the spring is allowed to come to its new equilibrium. It is then set in motion by stretching it a further 0.3m. The mass oscillates in simple harmonic motion

c) What is the period of the oscillation?

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5}{196}} = 1.00s$$

4. A 4.0 kg mass on a spring is stretched and released. The period of oscillation is measured to be 0.46 s. What is the spring constant?

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$0.46 = 2\pi\sqrt{\frac{4}{k}}$$
$$k = 746 N/m$$

5. A weight in a spring-mass system exhibits harmonic motion. The system is in equilibrium when the weight is motionless. If the weight is pulled down or pushed up and released, it would tend to oscillate freely if there were no friction. In a certain spring-mass system, the weight is 5 feet below a 10-foot ceiling when it is at rest. The motion of the weight can be described by the equation, y = 3sin(πt), where y is the distance from the *equilibrium point*, and t is measured in seconds.

   a) Find the period of the motion. 2 seconds.

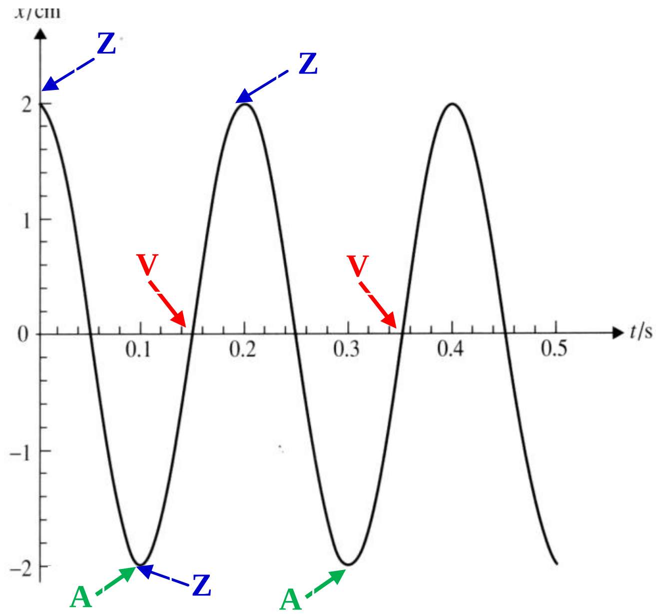
   The expression in the sin function, πt, is equal to 2πft. Therefore f = 0.5s and T = 2s

   b) What is the frequency of the motion? 1/2 cycle per second.

   c) Find the amplitude of the motion. 3ft

   d) How far from the ceiling is the weight after 3.5 seconds? 8 ft

6. The graph in the figure shows the displacement of a 0.040 kg particle from a fixed equilibrium position.



Use the graph to determine:

a) the period of motion   Period is time of 1 cycle = 0.2s

b) the maximum speed of the particle during oscillation,

The max speed is when the object passes through the equilibrium where there is no elastic potential energy. Rather all the energy is in the form of kinetic energy. The energy of the particle and spring is conserved since only the spring force (a conservative force) does work. We need to find the mechanical energy before finding the max speed.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$0.2 = 2\pi\sqrt{\frac{0.04}{k}}$$

$$k = 40N/m$$

$$E = E_A = ½kA^2 = ½ (40)(2^2) = 80J$$

$$E = E_0 = ½mv_0^2 = ½(0.04)v_B^2 = 80J$$

$$v_B = 63.2\ m/s$$

c) the maximum acceleration experienced by the particle.

Particle has maximum acceleration at max displacement from equilibrium, where $F_{net} = Fs_{pring}$ is greatest

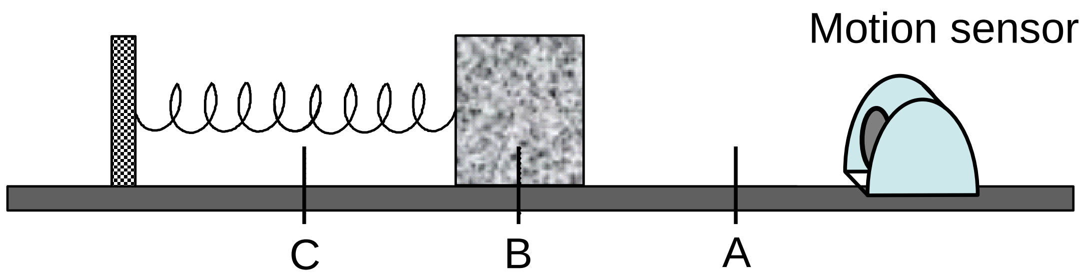
$$a_{max} = \frac{F_S}{m} = \frac{-k\Delta x_{max}}{m} = \frac{-(40)(2)}{0.04} = 2000m/s^2$$

On the graph, mark the following

a) a point where the velocity is zero (label this as Z)

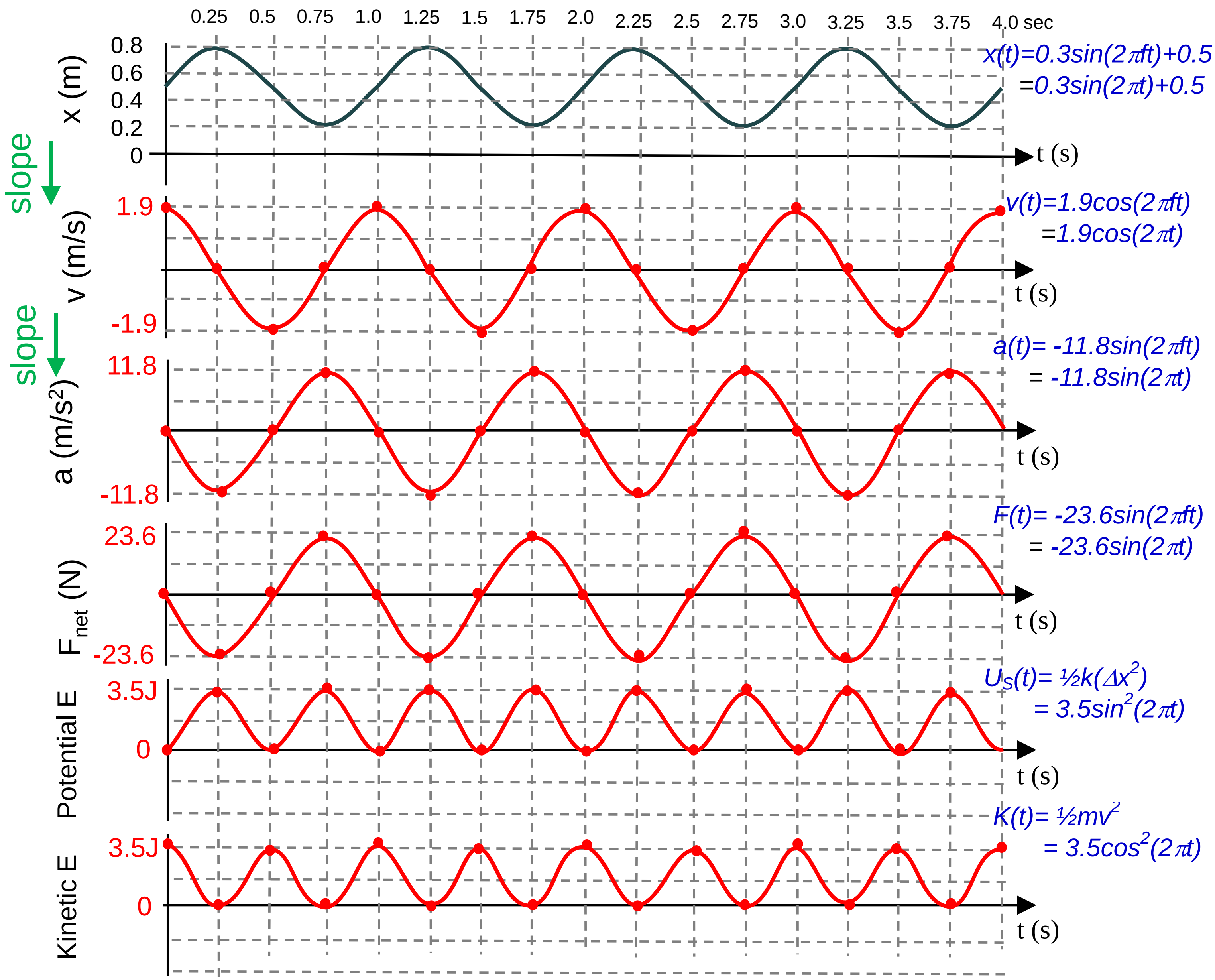
b) a point where the velocity is positive and has the largest magnitude (label this as V)

c) a point where the acceleration is positive and has the largest magnitude (label this as A).



7. The diagram above shows a 2 kg block attached to a Hookean spring on a <u>frictionless</u> surface. The block experiences no net force when it is at position B. When the block is to the left of point B the spring pushes it to the right. When the block is to the right of point B, the spring pulls it to the left.

   The mass is pulled to the left from point B to point A and released. The block then oscillates between positions A and C. A motion sensor placed to the right of position A gathers position-time data for the oscillating block. The position vs. time graph below describes the motion of this system for four cycles.



a) What is the period of oscillation for this system? T = 1 sec

b) What is the frequency of this oscillating system? f = 1/T = 1 Hz

c) $U_S(t) = ½k(\Delta x^2)$
$= 3.5\sin^2(2\pi t)$

d) What is the amplitude of vibration of this system? A = 0.3m

e) Determine the spring constant of the spring

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$
$$1 = 2\pi\sqrt{\frac{2}{k}}$$
$$k = 78.7N/m$$

f) Complete sketches for the other graphs shown based on the position vs. time graph. Label maxima and minima with numerical values on the axes.
Some things to consider when you sketch the graphs:
- You can make qualitative v-t and a-t sketches by considering the slopes of appropriate graphs
  To get a qualitative **v-t graph** (no numbers), take the slope of the x-t graph at points where it is zero, max or min and plot the v-t points shown. Connect points with a sinusoidal curve
  To get a qualitative **a-t graph** (no numbers), take the slope of the v-t graph at points where it is zero, max or min and plot the a-t points shown. Connect points with a sinusoidal curve
  F-t graph looks qualitatively like a-t since they are directly related by multiplying by the scalar *m*.
  Energy graphs are related to Δx^2 (potential) and v^2 (kinetic) so the negative regions on the x-t or v-t become positive on the energy graphs.
- Remember that $Us = ½ k\Delta x^2$ where $\Delta x = x - x_0$ is displacement from equilibrium ($x_0 = 0$ at the equilibrium position) Us is NOT $½ kx^2$ when $x_0 = 0$ at the motion sensor.
- Remember that $a = Fnet/m = (-kx)/m$

Show calculations necessary to label the graphs (max and min values of each variable)

Velocity Graph

$$v(t) = v_{\max}\cos(2\pi ft) = v_0\cos(2\pi ft)$$

v_{max} is at the equilibrium point where all the energy is kinetic energy. The energy of the mass-spring system is conserved at every position; $E = E_A = E_0 = E_x$

$$E_0 = E_A$$
$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kA^2$$
$$2v_0^2 = 78.7(0.3^2)$$
$$v_0 = 1.88m/s$$

Acceleration Graph

$$a(t) = -a_{\max}\sin(2\pi ft)$$

Max acceleration is at max displacement (Δx=A).

$$F_{net} = ma$$
$$k\Delta x = ma$$
$$kA = ma_{\max}$$
$$(78.7)(0.3) = 2a_{\max}$$
$$a_{\max} = 11.8m/s^2$$

Force Graph

$$F_{net} = ma$$
$$\max F_{net} = ma_{\max}$$
$$= 2(11.8) = 23.6N$$

Potential Energy Graph

U_S is proportional to Δx^2, not to x^2. $\Delta x = x - x_0$ where x_0 is the equilibrium position of the spring and block. From the x(t) graph, you can see that x_0 is 0.5m. Zero potential energy can be set anywhere. For convenience, define it to be the equilibrium point of the spring and block. Max U_S is at max Δx which is the amplitude (0.3m)

$$U_S = \tfrac{1}{2}k(\Delta x^2)$$

$$U_{S\max} = \tfrac{1}{2}k(A^2) = 3.54J$$

KineticEnergy Graph has same max as Potential E graph since energy is conserved

g) What is the average velocity of the block during one cycle?

$$\bar{v} = \frac{\Delta x}{T} = 0$$

h) What is the average speed of the block during one cycle?

In one cycle, the distance travelled can be seen on the x-t graph

$$\bar{s} = \frac{d}{T} = \frac{1.2}{1} = 1.2m/s$$

i) If the frequency of the oscillation were doubled, what would the average speed of the block be during one cycle? The average speed would double since the block would travel the same distance in ½ the time

$$\bar{s} = \frac{d}{T} = \frac{1.2}{0.5} = 2.4m/s$$