HW9.2: SHM-Springs and Pendulums

$$T_S = 2\pi\sqrt{\frac{m}{k}} \qquad T_P = 2\pi\sqrt{\frac{L}{g}}$$

Show your work clearly on a <u>separate page</u>. Make a sketch of the problem. Start each solution with a fundamental concept equation written in symbolic variables. Solve for the unknown variable in a step-by step sequence.

1. You want to build a grandfather clock with a pendulum (a weight on the end of a light cable) that has a one second period.

   a) How long do you make the cable?

   $$T = 2\pi\sqrt{\frac{L}{g}}$$

   $$1 = 2\pi\sqrt{\frac{L}{9.8}} \Rightarrow \quad L = 0.25m$$

   b) Suppose the cable stretches a bit from the weight on the end. Will the clock run fast or slow??

   slower, longer L gives larger period

2. What is the period of a simple pendulum 50 cm long

   a) on Earth

   $$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.50}{9.8}} = 1.42s$$

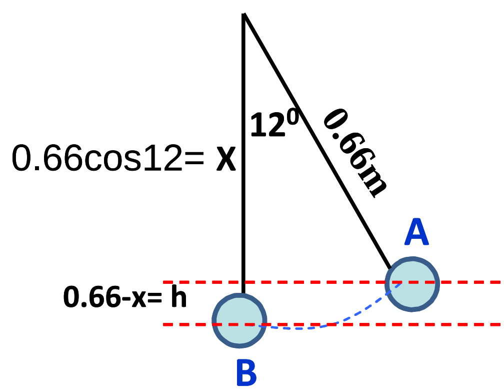
   b) on a freely falling elevator Freely falling elevator, g is apparently 0, T would be infinite

   c) on the Moon ($g_{Moon} = 1/6^{th} g_{Earth}$) if g is $1/6^{th}$ Earth, then T is √6 times T on Earth or 3.5s

3. The length of a simple pendulum is 0.66 m, the pendulum bob has a mass of 310 g, and it is released at an angle of 12^0 to the vertical.

   a) With what frequency does it oscillate? Assume SHM.

   $$f = 1/T = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8}{0.66}} = 0.613Hz$$



   b) What is the pendulum bob's speed when it passes through the lowest point of the swing? (Energy is conserved)

   $$E_A = E_B$$

   $$mgh = \tfrac{1}{2}mv_B^2$$

   $$(9.8)(0.66 - 0.66\cos 12) = \tfrac{1}{2}v_B^2$$

   $$v_B = 0.532m/s$$

   c) What is the total energy stored in the oscillation assuming no losses?

   $$E_A = E_B = \tfrac{1}{2}mv_B^2 = \tfrac{1}{2}(0.31)(0.532^2) = 0.0439J$$

4. Suppose you notice that a 5-kg weight tied to a string swings back and forth 5 times in 20 seconds. How long is the string?

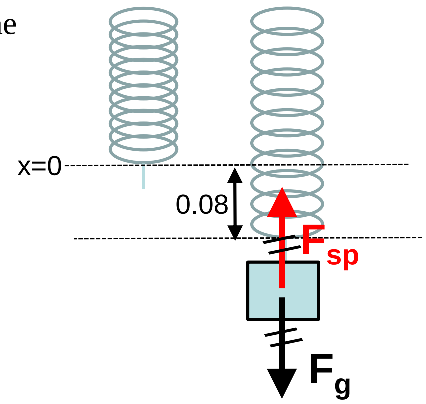
$$T = 2\pi\sqrt{\frac{L}{g}}$$

T = 20sec/5swings=4sec

$$4 = 2\pi\sqrt{\frac{L}{9.8}} \Rightarrow \quad L = 4.0m$$

5. The period of a pendulum is observed to be T. Suppose you want to make the period 2T. What do you do to the pendulum? Increase L by 4

6. (a) A mass of 400 g is suspended from a spring hanging vertically, and the spring is found to stretch 8.00 cm. Find the spring constant.

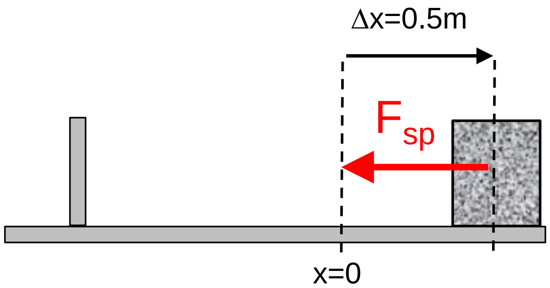


$$F_s = F_g$$
$$k\Delta x = mg$$
$$k = \frac{mg}{\Delta x} = \frac{(0.4)(9.8)}{0.08} = 49N/m$$

(b) How much will the spring stretch if the suspended mass is 575 g?

$$F_s = F_g$$
$$k\Delta x = mg$$
$$\Delta x = \frac{mg}{k} = \frac{(0.575)(9.8)}{49} = 0.115m$$

7. A 3.00-kg mass is attached to a spring and pulled out horizontally to a maximum displacement from equilibrium of 0.500 m.



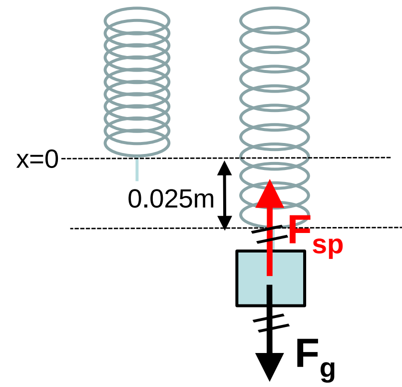
(a)What spring constant must the spring have if the mass is to achieve an acceleration equal to that of gravity?

$$\sum F = ma$$
$$F_s = ma$$
$$-k\Delta x = mg$$
$$k = \frac{mg}{\Delta x} = \frac{(3)(9.8)}{0.5} = 58.8N/m$$

(b) What is its period of vibration?

$$T_S = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3}{58.8}} = 1.42s$$

8. When a 4.00-kg mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg mass is removed, (a) how far will the spring stretch if a 1.50-kg mass is hung on it?



The amount that the spring stretches is directly proportional to the mass of the hanging object since the weight is balanced by the spring force. If 4 kg stretches the spring 2.5cm, then 1.5kg will stretch the spring (1.5/4)x2.5 cm = 0.94cm

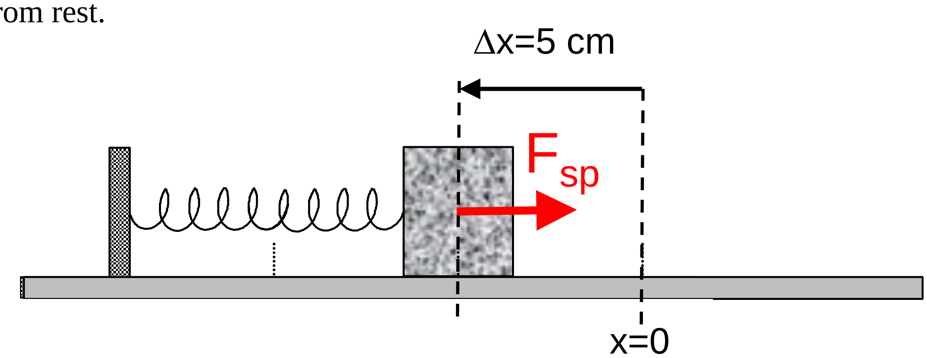
$$F_s = F_g$$
$$k\Delta x = mg$$
$$\Delta x = \frac{mg}{k}$$

b) How much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position? We need to determine the spring constant of this spring first.

$$F_s = F_g$$
$$k\Delta x = mg$$
$$k = \frac{mg}{\Delta x} = \frac{(4)(9.8)}{0.025} = 1568 N/m$$

$$W_s = \tfrac{1}{2}k(\Delta x^2) = \tfrac{1}{2}(1568)(0.04^2) = 1.25J$$

9. A spring of spring constant 19.6 N/m is compressed 5.00 cm. A mass of 0.300 kg is attached to the spring and released from rest.



Find (a) the maximum elastic potential energy stored in the spring

If there is no friction or air resistance, then the energy of the spring/mass is conserved. At the max displacement (at A), there is no kinetic energy and all of the energy is in the form of spring potential energy. Since we know k and A (5cm), we can find the energy at that point

$$E_A = U_{s\max} = \tfrac{1}{2}k(\Delta x)^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(19.6)(0.05^2) = 0.0245J$$

(b) the maximum speed of the mass and

Max speed and kinetic energy occurs at Δx=0, when $U_S = 0$. Then all of the 0.0245J of energy is in the form of kinetic energy.

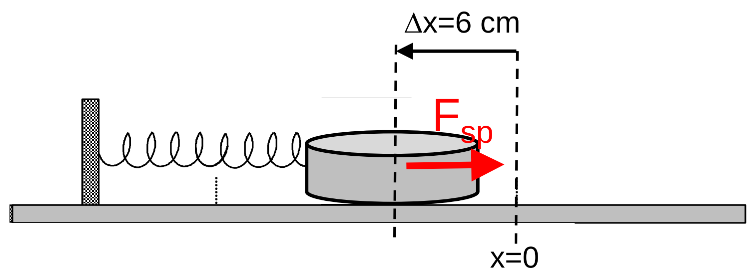
$$E_A = E_0 = 0.0245 = K_{\max} = \tfrac{1}{2}mv_{\max}{}^2 = \tfrac{1}{2}mv_0^2$$
$$v_0 = v_{\max} = 0.404 m/s$$

(c) the period of vibration of the mass.

$$T_S = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.3}{19.6}} = 0.78s$$

10. In an arcade game a 0.100-kg disk is shot across a frictionless horizontal surface by compressing it against a spring and releasing it. If the spring has a spring constant of 200 N/m and is compressed from its equilibrium position by 6.00 cm, find the speed with which the disk slides across the surface.

Since there are no nonconservative forces doing work on the hockey puck, the spring, earth and puck are an isolated system in which mechanical energy is conserved.



$$E_i = E_f$$

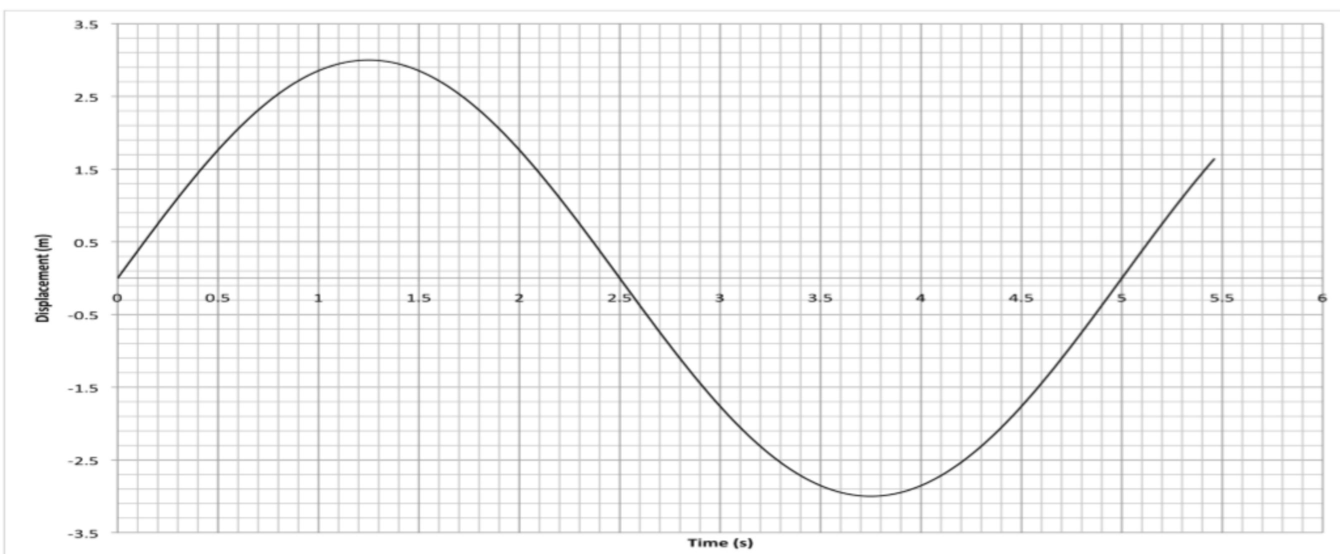
$$K_i + U_{si} = K_f + U_{sf}$$

$$0 + \tfrac{1}{2}k(\Delta x_i)^2 = \tfrac{1}{2}mv_f^2 + 0$$

$$\tfrac{1}{2}(200)(0.06^2) = 0.05v_f^2$$

$$0.36 = 0.05v_f^2$$

$$vf = 2.68m/s$$



11. The above graph shows the motion of a 12.0 kg object attached to a spring. The mass is undergoing simple harmonic motion. Determine the following…

a) The period of the motion 5s

b) The amplitude of the motion 3m

c) The spring constant

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$5 = 2\pi\sqrt{\frac{12}{k}} \Rightarrow k = 18.9N/m$$

d) The total mechanical energy in the mass-spring system E is the same at all points. At the max displacement (at A), there is no kinetic energy and all of the energy is in the form of spring potential energy. Since we know k and A (5cm), we can find the energy at that point

$$E_A = U_{s\max} = \tfrac{1}{2}k(\Delta x)^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(18.9)(3^2) = 85.1J$$

e) What is the maximum PE in the spring, and at what time(s) does it have this maximum energy?

U_{Smax}=85.1J - Max U_S when K = 0 at max displacements. This occurs at 1.25s and 3.75 s

f) What is the maximum KE of the object, and at what time(s) does it have this maximum energy?

K_{max}=85.1J - Max K when U_S = 0 at equilibrium. This occurs at 0s, 2,5s and 5 s

g) What is the maximum speed of the object, and at what time(s) did it occur?

Max speed occurs when K is max, at Δx=0 at 0s, 2.5s and 5s.

$$K_{\max} = 85.1 = \tfrac{1}{2}mv_{\max}{}^2 = \tfrac{1}{2}mv_0^2$$

$$v_0 = v_{\max} = 3.77m/s$$

h) What is the speed of the object when it is 1.50 meters from the equilibrium position?

At 1.50m, some of the 85.1J of energy is in the form of U_S and some is in form of K

$$E_{1.5} = 85.1 = K_{1.5} + U_{S1.5} = \tfrac{1}{2}mv_{1.5}^2 + \tfrac{1}{2}k(1.5^2)$$

$$85.1 = 6v_{1.5}^2 + 21.2625$$

$$v_{1.5} = 3.26m/s$$

i) What is the maximum acceleration of the object?

Max acceleration is where Fs is max which is at Δx=A

$$\sum F = ma$$

$$F_S = ma$$

$$-k\Delta x = ma$$

$$-kA = ma_{\max}$$

$$a_{\max} = \frac{kA}{m} = 4.73m/s^2$$

j) During what intervals in the first 5.00 seconds does the object have positive velocity?

Positive velocity occurs when the slope of the x-t graph is positive (0-1.25s and 3.75-5.5s)

k) During what intervals in the first 5.00 seconds is the object speeding up?

Speeding up when the displacement, velocity and acceleration are in the same direction.

0-1.25s: displ and velocity +direction / acceleration is - SLOWING DOWN

1.25-2.5s: displ and velocity -direction / acceleration is - SPEEDING UP

2.5-3.75s: displ and velocity -direction / acceleration is + SLOWING DOWN

3.75-5s: displ and velocity +direction / acceleration is + SPEEDING UP

5-5.5s: displ and velocity +direction / acceleration is - SLOWING DOWN

l) Determine the general equation for the displacement of the object as it relates to time. Use the appropriate numerical values for *A* and *f*. A is 3m and f is 1/5s

$$x(t) = A\sin(2\pi ft)$$

$$= 3\sin(\tfrac{2}{5}\pi t)$$

m) Determine the position of the mass at 3.13 s. The sine function must be evaluated in RADIANS

$$x(t) = 3\sin(\tfrac{2}{5}\pi t)$$

$$x(3.13) = 3\sin(\tfrac{2}{5}\pi(3.13)) = -2.13m$$

This result agrees with the x-t curve

12. Describe two experimental methods to determine the spring constant of a spring. Be specific about what you would measure, what equipment you would need to take your measurements, what are the independent and dependent variables. What you would graph and how you would analyze your graph to determine the spring constant.