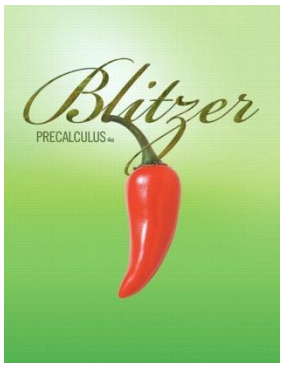
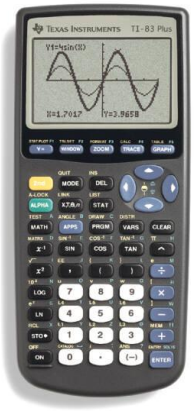


Pre-Calculus
Midterm Exam
REVIEW
January 2013



Name: _____ **Date:** _____

Teacher: _____ **Period:** _____

Your midterm examination will consist of:

- 40 multiple-choice questions (including true/false & matching) – these will be completed on the Scantron.
- 12 short answer questions – these will be completed in the test booklet. Show your work.
- 1 essay question in which you will be asked to explain a concept or describe your work.

The exam will cover material from Chapters 1, 2, and 3 and section 7.6.

NOTE: School policy mandates a penalty for cheating on an exam to be a grade of ZERO for that exam. The term cheating includes "intent to cheat." NO CELL PHONES. All cell phones must be kept out of sight. If a cell phone is seen during an exam, you will receive a grade of ZERO.

All calculators may be checked for inclusion of extraneous material. No papers should be placed in calculators. No information should be written on the front/back of calculators. The program portion of the graphing calculator will be checked. Any information entered there can be considered intent of cheating.

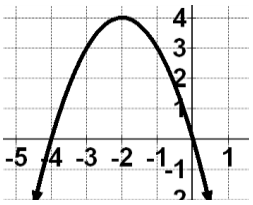
Before the examination, clear your calculator of any formulas, notes or any such items, which could be perceived as "useful" or providing unfair advantage. The best solution is to RESET and clear the memory completely.

The following pages provide a comprehensive review of the materials to be studied for this exam. We will take a few days of class time to review for this exam. Please feel free to stop in on your own time for further assistance. Good Luck!

Mr. Dominguez, Ms. Eisen, Mrs. Pogach, Mr. Hyman and Mr. Stella

1. Match the graph with the correct function.

a) $f(x) = -(x - 2)^2 + 4$
b) $f(x) = (x + 2)^2 + 4$
c) $f(x) = -2x + 4$
d) $f(x) = -(x + 2)^2 + 4$



2. Determine whether or not each equation defines y as a function of x.

a. $4x = y^2$

b. $xy - 4y = 4$

3. Determine whether each function is even, odd, or neither.

a) $f(x) = 2x^4 - 8x^5$

b) $f(x) = x\sqrt{5 - x^2}$

4. Evaluate h(7) if

$$h(x) = \begin{cases} x - 5, x \le 7 \\ 2x + 2, x \succ 7 \end{cases}$$

5. Find the equation of the axis of symmetry of the graph of: $y = 2x^2 - 8x - 7$

6. The vertex of the graph of $y = 3(x - 2)^2 + 1$ is:
a) (2, 1)
b) (–2, 17)
c) (4, 5)

7. If $y = 2x^2 - 12x + 20$, find the axis of symmetry and the coordinates of the vertex.

8. Graph the following piecewise function.

$$f(x) = \begin{cases} -.5x^2, x \prec 1 \\ 2x + 1, x \ge 1 \end{cases}$$

9. Find the domain $(f \circ g)(x)$ if $f(x) = \frac{5}{x + 4}$ and $g(x) = \frac{1}{x}$

10. Between what two successive integers does a real zero of $P(x) = x^3 - 2x - 5$ lie?

11. Find the domain of $f(x) = \frac{1}{\sqrt{x+5}}$

12. State each root and its multiplicity:
$(x^2 - 9)(x^2 + 2)(x^2 - 3)^2 = 0$

13. Solve: $x(x - 1)^2(x + 4) > 0$

14. Find the remainder when:
$x^{14} - 2x^5 + 5x^2 + 7$ is divided by $(x - 1)$

15. If *i* is a root of the equation
$x^4 - 3x^2 - 4 = 0$, find the other roots.

16. State the possible rational roots for:
$2x^3 - 3x^2 + x - 6 = 0$

17. On a certain route, a train line carries 7000 passengers per month each paying \$90. A market survey indicates that for each \$1 decrease in the ticket price, the airline will gain 60 passengers. Express the monthly revenue for the route, R, as a function of the ticket price, x.

18. Find the remainder using synthetic division:
$(x^4 - 5x^3) \div (x - 2)$

19. Solve: $x^4 - 5x^3 + 5x^2 + 5x - 6 = 0$

20. A rectangular dog park is to be fenced off into two adjacent rectangular dog runs, one for small dogs and one for big dogs. 600 feet of fencing is to be used. Find the dimensions that maximized the enclosed area and find the maximum area.

21. Solve completely: $2x^4 - x^3 - 3x^2 - 5x - 2 = 0$

22. Find the horizontal asymptote of the graph of:
$f(x) = \frac{9}{x+5}$

23. Find the vertical asymptotes, horizontal asymptotes, and the x-intercepts of the graph of:
$f(x) = \frac{x^2 - 2x - 3}{x^2 - 4}$

24. The zero(s) of $f(x) = \frac{x+5}{x^2 - 16}$ is (are):

25. Given that the polynomial equation $x^3 - 8x^2 + 17x - 10 = 0$ has 5 as a root, find the other two roots.

26. Graph & discuss multiplicity of the roots:
a) $y = (x - 1)(x + 3)(x - 2)^2$
b) $y = \frac{x-2}{x+5}$
c) $y = \frac{1}{x(x^2 - 4)}$
d) $y = x(x - 1)^3$
e) $y = -(x + 3)^2(x + 1)$

27. Sketch: $y = 2(x + 3)^2 - 2$

28. Graph the function given by the equation $f(x) = 4x - 3$ and its inverse f^{-1} on the same coordinate axes.

29. Sketch: $y = -(x - 3)^2$

30. Graph the function: $f(x) = |x - 2| + 2$

31. Let $f(x) = 2x^2 - x + 4$ and $g(x) = x^2 + 5$
Find: a) $(f + g)(x)$ b) $(f - g)(x)$
State in simplest form.

32. Let $f(x) = 2x^2 - 7$ and $g(x) = 3x + 1$
Find: a) $(f \circ g)(x)$ b) $(g \circ f)(x)$
State in simplest form.

33. Find the maximum or minimum value of the function $f(x) = 2x^2 - 3x + 4$. State whether this value is a maximum or minimum value.

34. Divide.

$$\frac{18x^4 + 9x^3 + 3x^2 + 3}{3x^2 + 1}$$

35. Simplify:
a) i^{1002}
b) $\sqrt{-16} + \sqrt{-25}$
c) $\dfrac{\sqrt{-2} + \sqrt{-5}}{\sqrt{-2} - \sqrt{-5}}$
d) $(3 - 4i) + (6 + 2i)$
e) $(3 - 4i) - (6 + 2i)$
f) $(3 - 4i)(6 + 2i)$
g) $\dfrac{3 - 4i}{6 + 2i}$

36. Use the rational root theorem to solve:

$$4x^4 - x^3 + 5x^2 - 2x - 6 = 0$$

Match each inequality in Column 1 with the graph of its solution set in Column 2.

Column 1	**Column 2**
37. $(x - 2)(x + 4) \le 0$	A. $(-\infty,-2) \cup (4, \infty)$
38. $(x + 2)(x - 4) > 0$	B. $(-2,4)$
39. $(x - 2)(x + 4) > 0$	C. $(-4,2)$
40. $(x + 2)(x - 4) < 0$	D. $(-\infty,-4) \cup (2, \infty)$
41. $(x + 2)(x - 4) \ge 0$	E. $[-4,2]$
42. $(x - 2)(x + 4) < 0$	F. $(-\infty,-2] \cup [4, \infty)$

43. Find $P(-2)$ if
$P(x) = -3x^4 + 7x^3 + 12x^2 + 21x + 9$

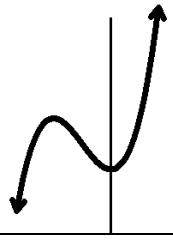
44. Which one of the following equations has two imaginary conjugate roots?
a) $3x^2 + 4x - 2 = 0$
b) $2 - 4x - 3x^2 = 0$
c) $-3x^2 + 4x + 2 = 0$
d) $3x^2 - 4x + 2 = 0$
e) $3x^2 - 4x - 2 = 0$

45. True or False:
The rational function $f(x) = \dfrac{(x+6)(x+7)}{x^2 - 36}$ has exactly 1 zero.

46. True or False:
A fourth-degree equation with real coefficients will always have 4 real roots.

47. How do you know that a cubic equation with real coefficients cannot have roots –1, –3, and 2 – i ?

48. The graph of a third degree polynomial function is given below. Sketch the x-axis so that the function has 2 real zeros, one with a multiplicity of 2.



49. What is the range of the function $g(x) = x^2 - 6x + 5$

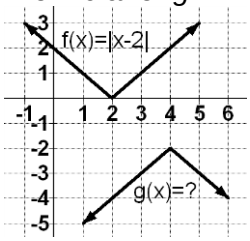
50. The synthetic division shown below illustrates the division of $2x^3 - 3x^2 - 10x + 5$ by d(x) with a remainder of r, where
a) d(x) = x + 3 and r = 2
b) d(x) = x – 3 and r = 2
c) d(x) = x + 3 and r = –2
d) d(x) = x – 3 and r = –2

3|	2	-3	-10	5
		6	9	-3
	2	3	-1	| 2

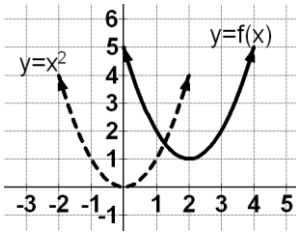
51. What is the domain of $f(x) = \frac{x-5}{\sqrt{x-2}}$?
a) set of all real numbers
b) set of all real numbers greater than 2
c) set of all real numbers except 2
d) set of all real numbers except 5
e) N.O.T.

52. Graph the equation $f(x) = 3^{x+1} - 4$; determine the domain and range. State in interval notation.

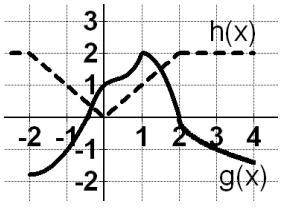
53. The graph of f(x) = |x – 2| is transformed to the congruent graph of *g* shown below. Write a formula for *g*.



54. Use the graph of $y = x^2$ to find a formula for the function y = f(x).
a) $f(x) = (x - 2)^2 + 1$
b) $f(x) = (x - 1)^2 + 2$
c) $f(x) = (x + 2)^2 + 1$
d) $f(x) = (x + 1)^2 - 2$
e) N.O.T.



55. Using the graphs of *g* and *h* below, find:
a) (g + h)(1)
b) (g – h)(–1)
c) (g + h)(0)
d) (g – h)(2)



56. Give the new coordinates if the point (8,-7) is reflected across:

a) the x-axis

b) the y-axis

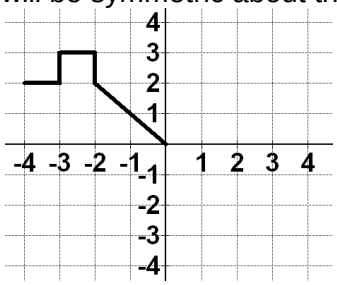
c) the origin

d) the line y = x

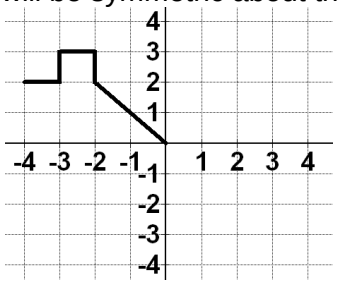
57. Find the value for each letter in this synthetic division process:

-2	3	8	0	4	20
		A	B	8	-24
	3	2	-4	C	D

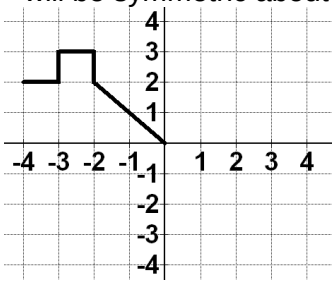
58. Complete the graph for $-4 \le x \le 4$ so that it will be symmetric about the x-axis.



59. Complete the graph for $-4 \le x \le 4$ so that it will be symmetric about the y-axis.



60. Complete the graph for $-4 \le x \le 4$ so that it will be symmetric about the origin.



61. Find the accumulated value of an investment of $25,000 over a period of 6 years at an interest rate of 2.3% compounded:

a) monthly

b) continuously

62. a) Rewrite $2 = \log_9 x$ in exponential form.

b) Rewrite $\sqrt[3]{64} = 4$ in logarithmic form.

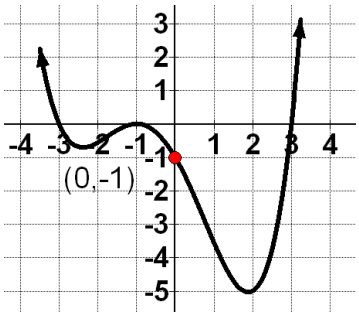
63. Explain.

a) $\log_b(de^5)$ b) $\ln\left[\dfrac{x^4\sqrt{x^2+5}}{(x-1)^7}\right]$

64. When your sink overflows, you call a plumber to "snake" the pipes. His fee varies linearly with the amount of time that he has to work. If he works for 20 minutes, the fee is $24. If he works for an hour, his fee is $32.

a) Define the variables, write the ordered pairs, find the slope, and find the particular equation of this function expressing dollars earned in terms of minutes worked.
b) What will his fee be for 40 minutes of work?
c) If the fee is $45, how long did he work?
d) What is the cost intercept and what is its real world meaning?
e) What are the units of the slope? What is its real world meaning?

65. Determine the equation of the curve drawn below.



66. Examine the graph of $y = 2x^2 – 3x – 1$ using your graphing calculator.

a) Copy the graph from your display.

b) Does the graph open up or down?

c) Does the graph have a maximum or minimum?

d) Use the CALC button to estimate the coordinates of the vertex.

e) Estimate the x-intercepts to the nearest hundredth (using CALC).

f) Using the formula, find the equation of the axis of symmetry.

g) Algebraically determine the vertex.

h) Using the quadratic formula, find the exact roots.

67. Using your graphing calculator, examine the graph of: $y = x^3 – 4x^2 + 3x + 7$

a) How many real roots do you see?

b) Estimate the real roots to the nearest hundredth.

68. Using your graphing calculator, examine the following graphs. In each case, state if the graph is **even, odd, or neither**. Explain what it means for a graph to be **even, odd, or neither**.

a) $y = –3x^4 + 2x^2 + 5$

b) $y = x^3 – 2x^2 –6x + 2$

c) $y = x^5 – 3x^3 + 2x$

69. Using your graphing calculator, examine the graph of: $f(x) = (x – 3)^2 (x – 1) (x + 2)$

a) What are the real zeros of this function?

b) Is there a double root? How do you know?

c) What is the range of this function?

d) Determine the intervals (of x) where $f(x) < 0$.

e) How would the graph of $–f(x)$ differ from the graph of $f(x)$?

70. Using your graphing calculator, examine the graph of: $f(x) = x^3 + 1$.

a) Sketch the graph.

b) Determine the inverse function of f(x).

c) Sketch the inverse function of f(x).

d) How do the two graphs compare?

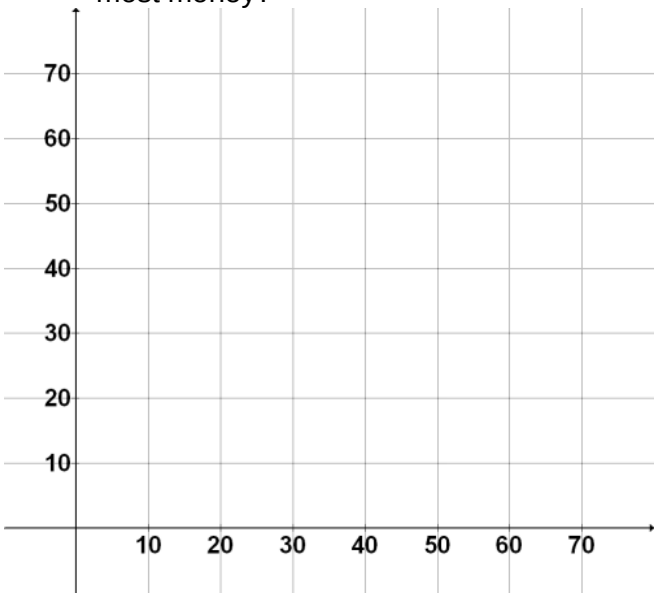
e) How do you know that the function has an inverse function?

71. How does the graph of $f(x) = 3|x + 2| – 1$ compare to the "parent" graph of $f(x) = |x|$? Discuss horizontal and vertical translations, stretches, or shrinks as they apply.

72. Craig Browning bakes cookies for the elementary school cookie sale. His chocolate chip cookies sell for $1.00 a dozen, and his oatmeal brownie cookies sell for $1.50 a dozen. He will bake up to 20 dozen chocolate chip cookies, and up to 40 dozen oatmeal brownie cookies, but no more than 50 dozen cookies total. Also, the number of oatmeal brownie cookies will be no more than three times the number of chocolate chip cookies. How many of each kind should Craig make in order for the elementary school to make the most money?

If x represents the number of dozen of chocolate chip cookies and y represents the number of dozen of oatmeal brownie cookies, graph the region to be maximized.

a) Determine the revenue function.
b) Shade the appropriate region on the graph that satisfies the constraints. Label all boundary lines and corner points.
c) How many of each kind should Craig make in order for the elementary school to make the most money?



73. The revenue equation of a company, in terms of the price of their product is:
$R = -15p^2 + 300p + 12000$.
Find the price that will yield the maximum revenue and determine the maximum revenue.

73. Write the expression as a single logarithm whose coefficient is 1.

$7 \ln x - 3 \ln y + \frac{1}{3} \ln z$

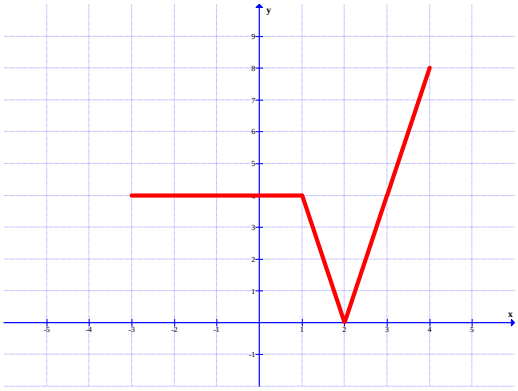
74. Solve: $8^{x+3} = 32^{x-1}$

75. Solve: $6^{-2x} = 12.8$
You may round to the nearest 1/100th.

76. Solve: $\log_2(x - 7) = 4$

77. a) Solve: $\log (x + 6) - \log 2 = \log (2x - 1)$
b) Solve: $3 \ln x = 12$

78. Use the following graph of f(x) to answer the questions below:



Sketch the following:
a) –f(x)

b) $\frac{1}{2}f(x)+1$

c) f(x-3)

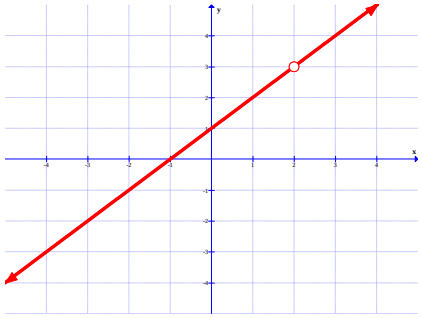
79. If $h(x)=[x]$ determine the transformation of h(x) if

a) $f(x)=[2x]$
b) $k(x)=3[x]$
c) $j(x)=[x]-1$

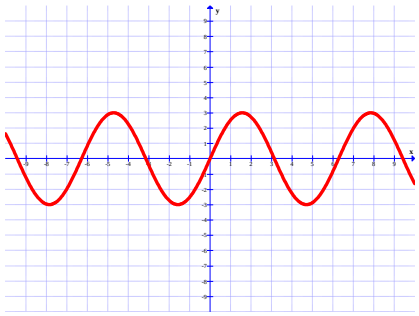
a) vertical stretch
b) vertical shrink
c) horizontal stretch
d) horizontal shrink
e) vertical shift up
f) vertical shift down

80. Determine the domain and range of the following graphs:

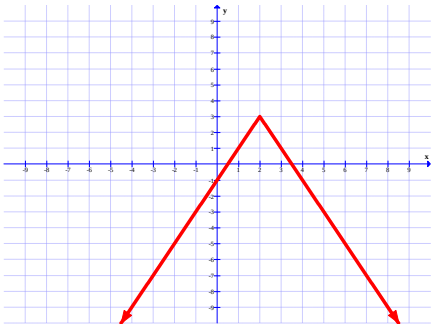
a)



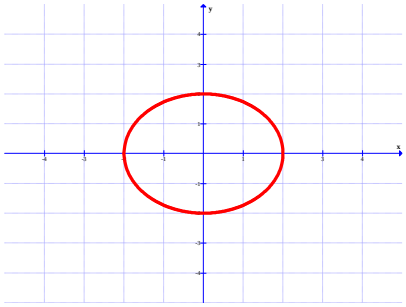
b)



c)



d)



81. Every day Rhonda needs a dietary supplement of 4 mg of vitamin A, 11 mg of vitamin B, and 100 mg of vitamin C. Either of two brands of vitamin pills can be used, Brand X at 6¢ or Brand Y at 8¢. Brand X supplies 2 mg of vitamin A, 3 mg of vitamin B and 25 mg of vitamin C. Brand Y supplies 1 mg of vitamin A, 4 mg of vitamin B and 50 mg of vitamin C. Write and graph a system of constraints to find how many of each pill Rhonda should take to minimize cost.

The following essay questions are comparable in difficulty to those on the midterm exam. They are not, however, the same questions.

Create a polynomial function of the sixth degree whose zeros have multiplicities of 1, 2, and 3, respectively. Explain your process.

Explain how you determine horizontal and vertical asymptotes in rational function graphs.

What is the discriminant and what role does it play in determining the nature of the roots in a quadratic equation. How does the nature of the roots affect the graph of the quadratic function?

Hannah solved the equation: $2x^3 + 6x^2 – 36x – 80 = 0$ and determined $x = –5$, $x = –2$, $x = 3$, or $x = 4$. How can you tell that she is incorrect?

Create a quadratic function and explain what is meant by vertically stretching and shrinking the graph of this function.

Over three years, would it be better to invest your money at 2% compounded weekly or 1.9% compounded continuously?

How is symmetry about the x-axis, y-axis, and the origin defined?

How do you determine end behavior, domain and range of a polynomial function?

ANSWERS

1. D
2. $\left(\frac{5}{9}, \frac{1}{3}, \frac{4}{9}\right)$
3. A) –4
   B)) –1
   C) 9
   D) 6
4. $-\frac{1}{5}$
5. X = 2
6. a
7. x = 3, (3, 2)
8. $-\frac{3}{2}, -\frac{3}{2}, c = \frac{27}{2}$
9.

Pos.Real	NegReal	Imag
2	2	0
2	0	2
0	2	2
0	0	4

10. 2< x < x
11. 7
12.

Root	Multiplicity
3	1
-3	1
$i\sqrt{2}$	1
$-i\sqrt{2}$	1
$\sqrt{3}$	2
$-\sqrt{3}$	2

13. x<–4 or 0<x< 1 or x > 1
14. 11
15. –i, 2, –2
16. $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 6$
17. Sum =3 , product = 3
18. –24
19. 3, 1, 2, –1
20. 3 < x < 4
21. $-\frac{1}{2}, 2, -\frac{1}{2} \pm \frac{1}{2} i\sqrt{3}$
22. Y = 0
23. V.A.: x = 2, x = –2
    H.A.: y =1
    x-int.: 3, –1
24. –5
25. 2, 1
26. a) f(x)=(x-1)*(x+3)*(x-2)^2

    b) f(x)=(x-2)/(x+5)

    c) f(x)=1/(x*(x^2-4))

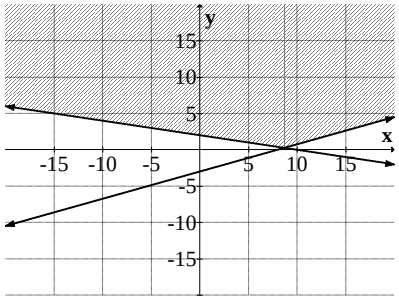
    d) f(x)=x*(x-1)^3

    e) f(x)=-(x+3)^2*(x+1)
27. f(x)=2*(x+3)^2-2
28. f(x)=4x-3
    f(x)=(x+3)/4
    line of reflection
29. f(x)=-(x-3)^2
30. f(x)=abs(x-2)+2
31. a) $3x^2 - x + 9$
    b) $x^2 - x - 1$
32. a) $18x^2 + 12x - 5$
    b) $6x^2 - 20$
33. Minimum value is $\frac{23}{8}$

34. a) (4 , -1)
b) 7x -3y = 31

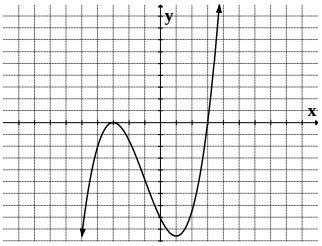
35. a) -1 f) 26 – 18i
b) 9i g) $\frac{1}{4}-\frac{3}{4}i$
c) $\frac{-7-2\sqrt{10}}{3}$
d) 9 – 2i
e) -3 – 6i

36.

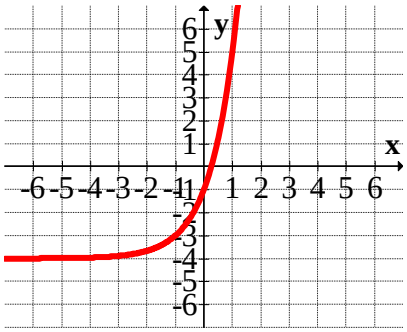


37. E
38. A
39. D
40. B
41. F
42. C
43. -91
44. D
45. True
46. False
47. Because if 2 – i is a root then 2 + i must also be a root, since complex roots come in conjugate pairs. Then there would be 4 roots not three. The equation there fore cannot be a cubic.

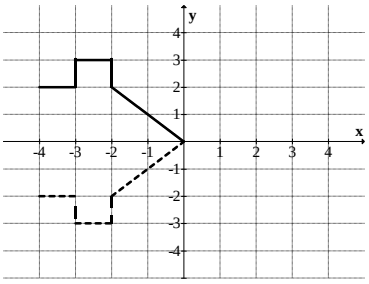
48.



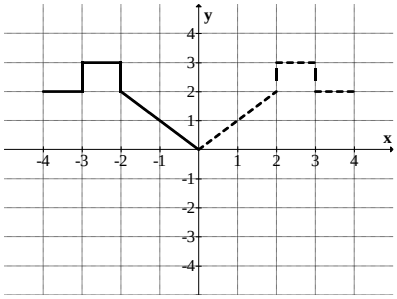
49. Range: {y : y ≥ -4}
50. b
51. b
52. D : {x : -2 ≤ x ≤ 4}
R : {y : -3 ≤ y ≤ 2}



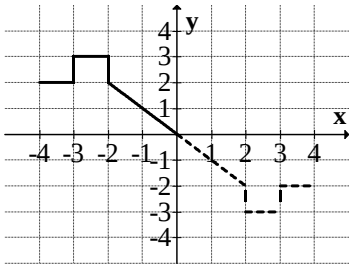
53. $g(x)=-|x-4|-2$
54. a
55. a) 3 b) -2 c) 1 d) -2
56. False
57. A = -6 B = -4
C = 12 D = -4
58.



59.



60.



61. a) $28695.60
b) $28,699.39

62. a) $9^2 = x$
b) $\log_{64} 4 = \frac{1}{3}$

63. a) $\log_b d + 5\log_b e$
b) $4\ln x + .5\ln(x^2+5) - 7\ln(x-1)$

64.
a) Let x = no. of minutes worked
Let y = no. of dollars earned
(20 , 24) and (60 , 32)
Slope is $\frac{1}{5}$
$y=\frac{1}{5}x+20$
b) $28
c) 125 minutes
d) $20 service charge
e) $1 per 5 minutes intervals or 20¢ per minute

65.
$y=\frac{1}{9}(x+3)(x+1)^2(x-3)$

66. a) f(x)=2x^2-3x-1
b) up
c) minimum point
d) (0.8 , -2.12)
e) (-0.28 , 0) & (1.78 , 0)
f) $x=\frac{3}{4}$
g) $\left(\frac{3}{4},-\frac{17}{8}\right)$
h) $x=\frac{3\pm\sqrt{17}}{4}$

67. a) one b) x = -0.93

68. a) even b) neither
c) odd
An ***even*** function is symmetric with respect to the y-axis; whenever (x , y) is a point on the graph, so is (-x , y); f(x) = f(-x), for all x in the domain.
An odd function is symmetric with respect to the origin; whenever (x , y) is a point on the graph, so is (-x , -y); f(-x) = -f(x) for all x in the domain.
Neither means that the function is neither even nor odd.

69.
a) -2, 1, 3
b) Yes at x = 3 there is a turning point or relative minimum point.
c) $y \geq -32$
d) $-2 < x < 1$
e) if(x) would "flip" or reflect the graph of f(x) over the x-axis.

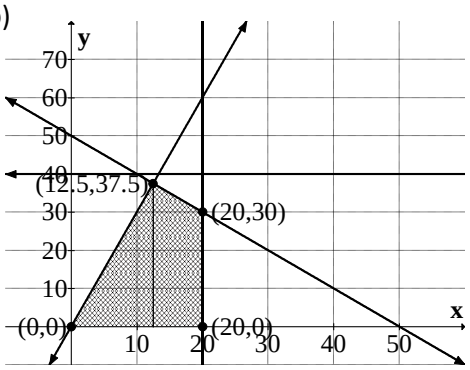
70. a) f(x)=x^3+1

b) $f^{-1}(x) = \sqrt[3]{x-1}$
c) f(x)=(x-1)^(1/3)

d) The two graphs are reflections over the line y = x.
e) f(x) passes the horizontal line test

71. The graph of $f(x) = 3|x+2| - 1$ vertically stretches the graph of $f(x) = |x|$ and translates or shifts the graph horizontally 2 units to the left and vertically 1 unit down.

72. a) $P(x,y) = x + 1.5y$
b)



Constraints:
$0 \leq x \leq 20$
$0 \leq y \leq 40$
$x + y \leq 50$
$y \leq 3x$

Corner Points:
$(0,0)$
$(12.5,37.5)$
$(20,30)$
$(20,0)$

c) 12 ½ dozen of choc. chip
7 ½ dozen of oatmeal brownie

73. price: $10
Max. rev.: $13,500

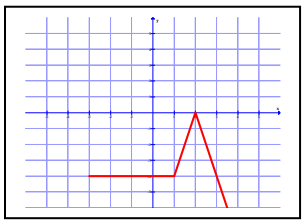
74. $ln \frac{x^7 \sqrt[3]{z}}{y^3}$

75. x=7

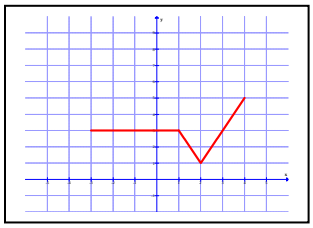
76. x=7.11

77a) x=23
b) $x = e^4 \approx 54.598$

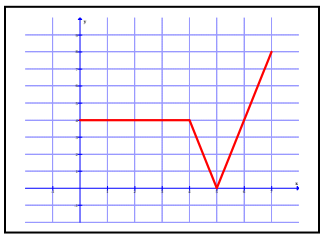
78.a)



b)



c)

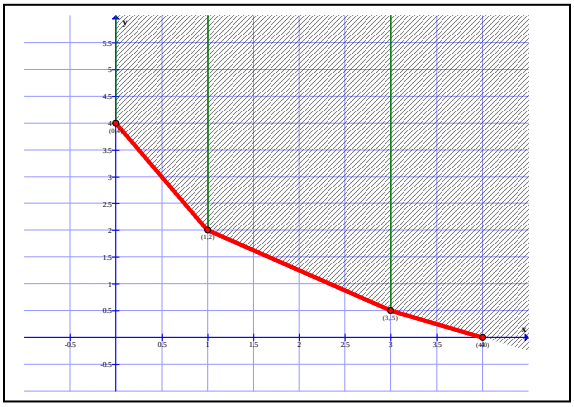


79.a) d
b) a
c) f

80. a) D: $(-\infty, 2) \cup (2, \infty)$
R: $(-\infty, 3) \cup (3, \infty)$
b) D: $(-\infty, \infty)$
R: $[-3,3]$
c) D: $(-\infty, \infty)$
R: $(\infty,3]$
d) D: $[-2,2]$
R: $[-2,2]$

81.

	Brand X (mg)	Brand Y (mg)	Min.
Vit. A	2	1	4
B	3	4	11
C	25	50	100
Cost	6¢	8¢	



Constraints:

$x \geq 0$

$y \geq 0$

$2x + y \geq 4$

$3x + 4y \geq 11$

$x + 2y \geq 4$

Corner Points:

$(0,4) = 32$

$(1,2) = 22$

$(3,0.5) = 22$

$(4,0) = 24$

c) 1 Brand X pill and 2 Brand Y pills