Name______________________________ Date__________

2D Motion WS

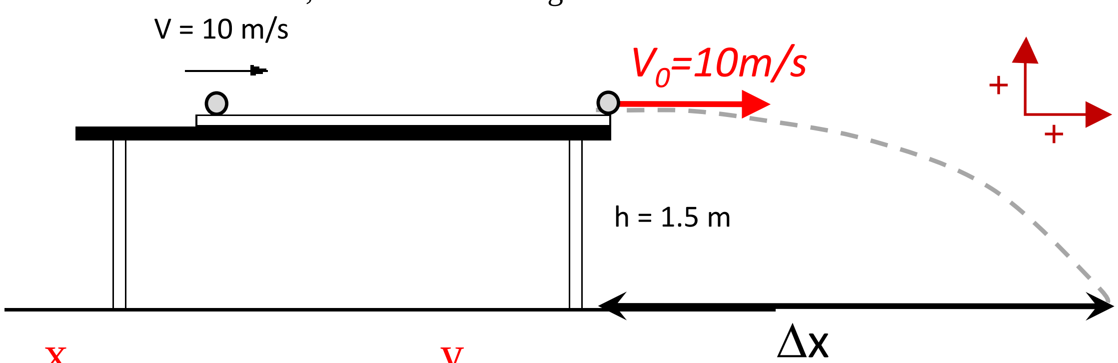
The equations of motion that relate to projectiles were discussed in the Projectile Motion Analysis Activity. You found that a projectile moves with constant velocity in the horizontal direction and constant acceleration in the vertical direction (a_y = -9.8 m/s^2). You can use the same equations from the previous unit to solve projectile motion problems keeping in mind horizontal motion is independent of vertical motion. Use separate sheets to solve problems. ***Show all work including a diagram of the problem, list of x- and y- variables that indicate initial and final conditions, and the equations you use to solve the problem***.

Horizontally Launched Projectiles

A horizontally launched projectile's <u>initial vertical velocity is zero</u>. Solve the following problems with this information.

1. Given the following situation of a marble in motion on a rail with negligible friction

   a. Once the ball leaves the table, calculate how long it will take for the ball to hit the floor.



<u>x</u>

$$v_x = v_0 = 10m/s$$
$$\Delta x =$$
$$t =$$

<u>y</u>

$$v_{0y} = 0$$
$$v_{fy} =$$
$$\Delta y = -1.5m$$
$$a_y = -9.8m/s^2$$
$$t =$$

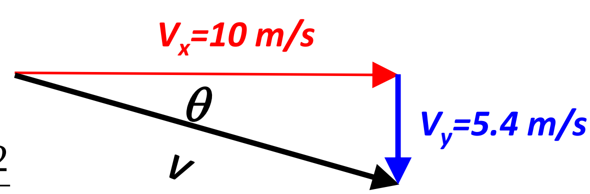
$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$
$$\boxed{\sqrt{\frac{2\Delta y}{a_y}} = t}$$
$$0.553s = t$$

   b. Determine the impact <u>velocity</u> (magnitude and direction) of the marble right before it hits the floor

   We need to determine both the x- and y- components of the impact velocity and then put them together to find the 2-D impact velocity vector.

   v_x does not change so we already know that (v_x = 10 m/s)

$$v_y = v_{0y} + a_y t$$
$$= 0 - 9.8(0.553) = -5.42m/s$$
$$v^2 = v_x^2 + v_y^2$$
$$v = 11.4m/s$$



$$\tan\theta = \frac{v_y}{v_x} = \frac{5.42}{10}$$
$$\theta = 28.5^o$$

v = 11.4 m/s, 28.5^o below the horizontal

c. How far will the ball travel horizontally before hitting the floor?

$$\begin{aligned}\Delta x &= v_x t \\ &= 10(0.553) \\ \Delta x &= 5.53m\end{aligned}$$

If the table were 3.0 m high (so we have doubled the height), and sphere was traveling with the same velocity of 10 m/s while on the table, determine each of the following....

d. Determine how much longer it takes the marble to fall to the floor.

Relationship between Flight Time and height (constants ay and v_{0y}): see boxed equation in a)

$$t = \sqrt{\frac{2\Delta y}{g}}$$

If the height of the table is doubled, flight time increases by $\sqrt{2}$

e. What effect did doubling the height have on range of the marble? What other factors affect the range of the sphere?

Range depends on v_x (which is unchanged) and flight time

$$\Delta x = v_x t$$

So, if the height of the table is doubled, range increases by $\sqrt{2}$

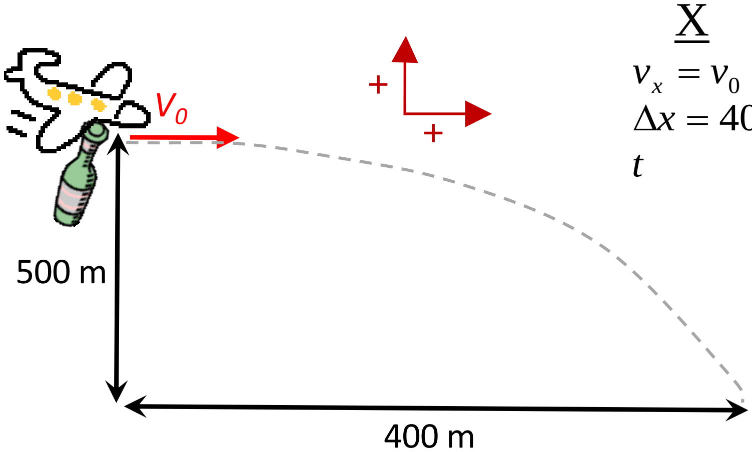
range of 3m high table is $\sqrt{2}(5.53) = 7.82m$

<u>other factors that affect the range</u>:

initial velocity (if doubled, then range doubled)

acceleration due to gravity (if doubled, range reduced by $1/\sqrt{2} = 0.71$)

2. A bottle is dropped from a moving airplane (ignore the effect of air resistance). If the plane from which the bottle was dropped was flying at a height of 500m, and the bottle lands 400m horizontally from the initial dropping point,



X	Y
$v_x = v_0 =$	$v_{0y} = 0$
$\Delta x = 400m$	v_y
t	$\Delta y = -500m$
	$a_y = -9.8m/s^2$
	t

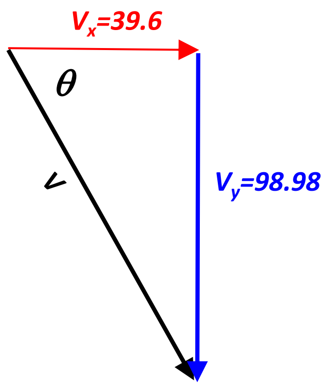
a) how fast was the plane flying when the bottle was released?

$$\begin{aligned}\Delta y &= v_{0y}t + \tfrac{1}{2}a_y t^2 \\ -500 &= 0 - 4.9t^2 \\ t &= 10.1s\end{aligned}$$

$$\begin{aligned}\Delta x &= v_x t \\ 400 &= v_x(10.1) \\ v_x &= v_0 = 39.6m/s\end{aligned}$$

b) what was the velocity of the bottle right before it hit the ground?

$$v_y = v_{0y} + a_y t = 0 - 9.8(10.1) = -98.98 m/s$$

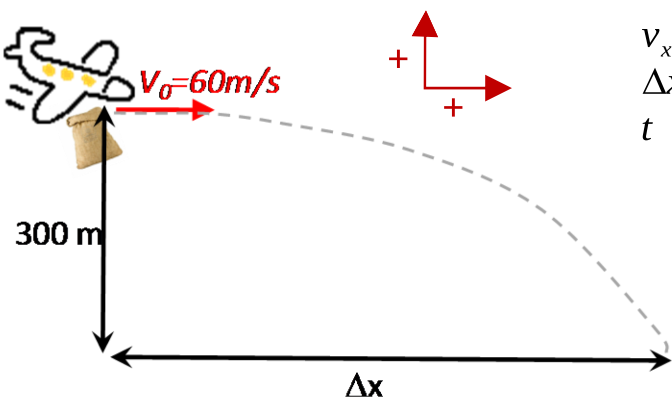


$$v^2 = v_x^2 + v_y^2$$
$$v = 106.6 m/s$$

$$\tan\theta = \frac{98.98}{39.6}$$
$$\theta = 68.2^o$$

v = 106.6 m/s, 68.2^o below the horizontal (-68.2^o from x-axis)

3. Suppose that an airplane flying 60 m/s, at a height of 300m, dropped a sack of flour (ignore the effect of air resistance).



X	Y
$v_x = v_0 = 60$	$v_{0y} = 0$
$\Delta x =$	v_y
t	$\Delta y = -300m$
	$a_y = -9.8 m/s^2$
	t

a) How far from the point of release would the sack have traveled when it struck the ground?

$$\Delta y = v_{0y}t + \tfrac{1}{2} a_y t^2$$
$$-300 = 0 - 4.9t^2$$
$$t = 7.82s$$

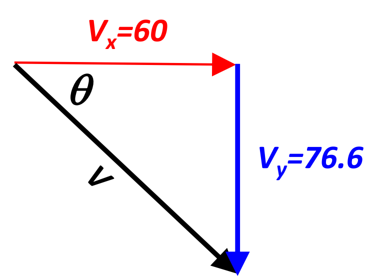
$$\Delta x = v_x t$$
$$= 60(7.82)$$
$$\Delta x = 469m$$

b) What would be the impact velocity of the sack of flour?

$$v_y = v_{0y} + a_y t = 0 - 9.8(7.82) = -76.64 m/s$$

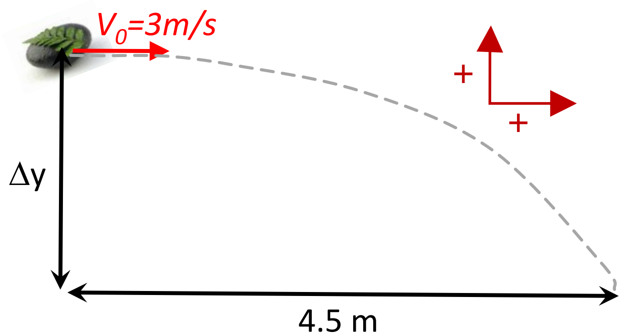
$$v^2 = v_x^2 + v_y^2$$
$$v = 97.3 m/s$$

$$\tan\theta = \frac{76.6}{60}$$
$$\theta = 52^o$$



v = 97.3 m/s, 52^o below the horizontal

4. In many locations, old abandoned stone quarries have become filled with water once excavating has been completed. While standing on a quarry wall, a boy tosses a piece of granite into the water below. If he throws the ball horizontally with a velocity of 3.0 m/s, and it strikes the water 4.5 m away, how high above the water is the wall? (ignore the effect of air resistance)

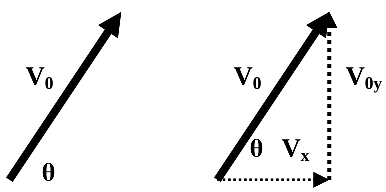


X	Y
$v_x = v_0 = 3m/s$	$v_{0y} = 0$
$\Delta x = 4.5m$	v_y
t	$\Delta y =$
	$a_y = -9.8m/s^2$
	t

$$\Delta x = v_x t \qquad \Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$
$$4.5 = 3t \qquad = 0 - 4.9(1.5^2)$$
$$t = 1.5s \qquad \Delta y = -11.0m$$

Projectiles Launched at an Angle

Projectile motion and vectors



$$\boxed{v_x = v_0 \cos\theta} \qquad \boxed{v_{0y} = v_0 \sin\theta}$$

- A projectile's velocity (v) has an X component (v_x) and a Y component (v_y). The X component (v_x) is found by multiplying the magnitude of the velocity by the ***cosine*** of the angle, θ.
- Similarity, the Y component of velocity is found by multiplying the magnitude of the velocity by the ***sine*** of the angle, θ.

So, a projectile fired at **20 m/s** at **65°** has an X-velocity of $\boxed{v_x = 20\cos 65}$ or **8.5** m/s.

The projectile would have a Y-velocity of $\boxed{v_{0y} = 20\sin 65}$ or **18** m/s.

So, the projectile would fire as far as one fired horizontally at 8.5 m/s and as high as one fired straight up at 18 m/s.

5. A lacrosse player slings the ball at an angle of 30 degrees above the horizontal with a speed of 20 m/s.

~~x~~ y

$$v_x = v_0 \cos\theta$$
$$= 20\cos 30 = 17.32 m/s$$
$$\Delta x =$$
$$t =$$

$$v_{0y} = v_0 \sin\theta$$
$$= 20 \sin 30 = 10 m/s$$
$$v_y =$$
$$\Delta y = 0$$
$$a_y = -9.8 m/s^2$$
$$t =$$

a) How far away should a teammate be in order to catch the ball at the same height it was released?

$$\Delta y = v_{0y}t + \tfrac{1}{2} a_y t^2$$
$$0 = 10t - 4.9t^2$$
$$t = 2.04s$$

$$\Delta x = v_x t$$
$$= 17.32(2.04)$$
$$\Delta x = 35.3m$$

b) What is the velocity of the ball right before it is caught?

Since the ball lands at the same height as it was launched, time up is equal to time down. Therefore, final ***speed*** is equal to initial ***speed***; direction of final velocity below the horizontal the same amount as the initial was above.

v = 20 m/s, 30^0 below the horizontal (-30^0 from x-axis)

c) What is the ball's maximum height? Conceptually, ball travels up for 1s at average vertical velocity of 5m/s ($v_{av\,y}$=1/2 (v_{0y}+v_y)), so it goes to a max height of $\Delta y = v_{avy}t$ = 5m/s

<u>x</u>

$$v_x = 20\cos 30 = 17.32 m/s$$
$$\Delta x =$$
$$t =$$

<u>y</u>

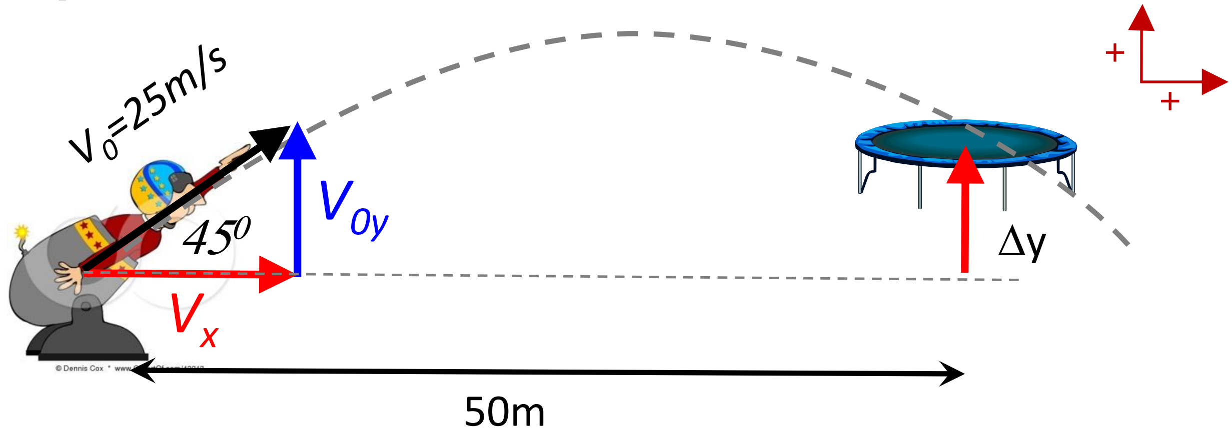
$$v_{0y} = 20 \sin 30 = 10 m/s$$
$$v_y = 0$$
$$\Delta y_{\max} =$$
$$a_y = -9.8 m/s^2$$
$$t =$$

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y_{\max}$$

$$\Delta y_{\max} = \frac{-v_{0y}^2}{2a_y} = \frac{0-100}{-19.6} = 5.1m$$

c) What is the ball's velocity at the peak? $\boldsymbol{v_y}$ = 0 at peak, $\boldsymbol{v_x}$=17.32 m/s. Therefore, $\boldsymbol{v_{peak}}$ = 17.32m/s, in +x direction

6. A daredevil is shot out of a cannon at an angle of 45° with an initial speed of 25 m/s. A net is positioned at a horizontal distance of 50 m from the cannon. At what height above the cannon should the net be placed in order to catch the daredevil?



X

$$v_x = v_0 \cos\theta$$
$$= 25\cos 45 = 17.7 m/s$$
$$\Delta x = 50m$$
$$t =$$

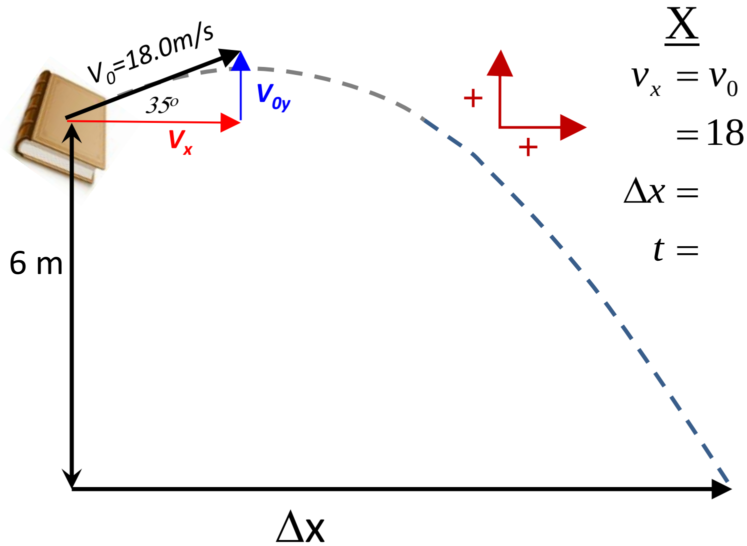
Y

$$v_{0y} = v_0 \sin\theta$$
$$= 25\sin 45 = 17.7 m/s$$
$$v_y =$$
$$\Delta y =$$
$$a_y = -9.8 m/s^2$$
$$t =$$

$$\Delta x = v_x t$$
$$50 = 17.7t$$
$$t = 2.82s$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$
$$= 17.7(2.82) - 4.9(2.82^2)$$
$$\Delta y = +10.9m$$

7. Frustrated with the book you're reading, you open the second story classroom window and violently hurl your book out the window with a velocity of 18 m/s at an angle of 35 degrees above the horizontal. If the launch point is 6 meters above the ground,



X

$$v_x = v_0 \cos\theta$$
$$= 18\cos 35 = 14.7$$
$$\Delta x =$$
$$t =$$

Y

$$v_{0y} = v_0 \sin\theta$$
$$= 18\sin 35 = 10.3$$
$$v_y =$$
$$\Delta y = -6m$$
$$a_y = -9.8 m/s^2$$
$$t =$$

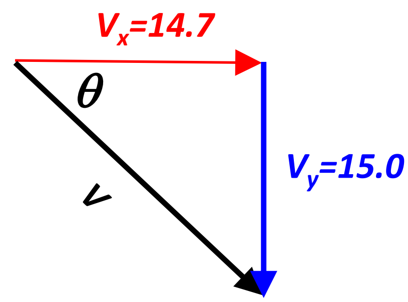
a) how far from the building will the book hit the parking lot?

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$
$$-6 = 10.3t - 4.9t^2$$
$$0 = -4.9t^2 + 10.3t + 6$$
$$t = \boxed{2.58s}\ or\ -0.48s$$

$$\Delta x = v_x t$$
$$= 14.7(2.58)$$
$$\Delta x = 37.9m$$

b) What will the impact velocity of the book be?

$$v_y = v_{0y} + a_y t = 10.3 - 9.8(2.58) = -15.0m/s$$

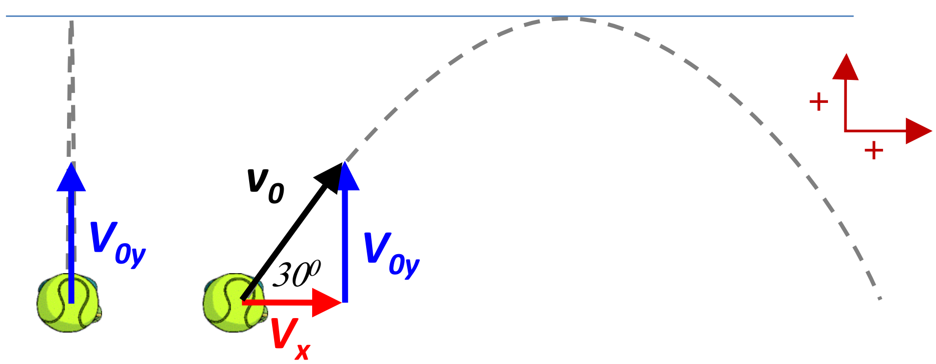


$$v^2 = v_x^2 + v_y^2$$
$$v = 21m/s$$

$$\tan\theta = \frac{v_{fy}}{v_x} = \frac{15}{14.7}$$
$$\theta = 45.6^o$$

v_f = 21 m/s, 45.6^0 below the horizontal (-45.6^0 from x-axis)

8. A ball is thrown straight upward and returns to the thrower's hand after 3 seconds in the air. A second ball is thrown at an angle of 30 degrees with the horizontal. At what speed (remember that this is the resultant magnitude of the vertical and horizontal speeds) must the second ball be thrown so that it reaches the same height as the one thrown vertically?



Since the vertical motion of the projectile is independent of the horizontal component, the projectile's vertical component must be identical to the free fall. To reach the same height as the ball thrown straight up, the <u>projectile and the straight up ball must have the same initial vertical velocities. OR they must be in air for the same time (3 sec). Since they must both reach peak in 1.5s (and loose all vertical velocity in 1.5s), the initial vertical velocity must be 15m/s</u>

You could also find the initial vertical velocity using kinematic equations:

<u>Ball up</u>

$$v_{0y} =$$
$$v_y =$$
$$\Delta y = 0$$
$$a_y = -10m/s^2$$
$$t = 3s$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$
$$0 = v_{0y}(3) - 5(9)$$
$$v_{0y} = 15m/s$$

Projectile Ball

With the angle and v_{0y}, can use trig to find v_0:

$$\sin 30 = \frac{v_{0y}}{v_0}$$

$$v_0 = \frac{v_{0y}}{\sin 30} = \frac{15}{0.5} = 30 m/s$$