Gravitation and Orbital Motion

1. A neutron star and a black hole are 3.34 x 10^{12} m from each other at a certain point in their orbit. The neutron star has a mass of 2.78×10^{30} kg and the black hole has a mass of 9.94×10^{30} kg.
   a) What is the magnitude of the gravitational attraction on the neutron star due to the black hole?

$$F_G = \frac{GM_{NS}M_{BH}}{R^2} = \frac{(6.67x10^{-11})(2.78x10^{30})(9.94x10^{30})}{(3.34x10^{12})^2} = 1.65x10^{26}N$$

   b) What is the gravitational force on the black hole?
   It is equal (1.65 x 10^{26} N) and opposite in direction

2. a) What is the force of gravity on a 24.0 kg table? Fg = weight. On earth, Fg = mg = 24(9.8) = 235N
   b) What would happen to the force of gravity if the distance between the table and the center of the earth was tripled? Fg would go down 9 times to 26N halved? Fg would go down 4 times to 58.8N

3. An asteroid orbiting the Sun has a mass of 4.00×10^{16} kg. At a particular instant, it experiences a gravitational force of 3.14×10^{13} N from the Sun. The mass of the Sun is 1.99×10^{30} kg. How far is the asteroid from the Sun?

$$F_G = \frac{GM_{Sun}m_{asteroid}}{R^2}$$

$$3.14x10^{13} = \frac{(6.67x10^{-11})(1.99x10^{30})(4.00x10^{16})}{R^2}$$

$$R = 4.11x10^{11}m$$

4. A planetoid has a mass of 2.83 x 10^{21} kg and a radius of 7.00×10^5 m. Find the magnitude of the gravitational acceleration (or gravitational field strength) at the planetoid's surface. At the planetoid's surface, an object would be a distance of $R_{planetoid}$ from the center of the planetoid where $R_{planetoid}$ is the radius of the planetoid.

$$F_G = mg = \frac{GM_{planetoid}m}{R_{planetoid}^{\ 2}}$$

at surface

$$g = \frac{F_G}{m}$$

at surface $$g = \frac{GM_{planetoid}}{R_{planetoid}^{\ 2}} = \frac{(6.67x10^{-11})(2.83x10^{21})}{(7x10^5)^2} = 0.385m/s^2$$

5. If you were to travel across the solar system and land on different planets, you would notice that you weighed different amounts on the different planets. This is because the gravitational field (acceleration due to gravity, surface gravity) on their surfaces is different from one another.
   Use the information in the table below to fill in the blanks. The mass of the Sun is 1.99×10^{30} kg

Object	Mass of Object (kg)	Radius of Object (m)	Distance to Sun (m)	F_G between Sun and Object (N)	Surface Gravitational Field Strength (N/kg)	Weight of an 80kg student on the object's surface (N)
Moon	7.36×10^{22}	1.74×10^{6}	1.50×10^{11}	4.34×10^{20}	1.62	130
Venus	4.87×10^{24}	6.05×10^{6}	1.08×10^{11}	5.54×10^{22}	8.87	710
Earth	5.98×10^{24}	6.37×10^{6}	1.50×10^{11}	3.52×10^{22}	9.83	786
Mars	6.40×10^{23}	3.40×10^{6}	2.28×10^{11}	1.63×10^{21}	3.69	295
Jupiter	1.90×10^{27}	7.15×10^{7}	7.78×10^{11}	4.17×10^{23}	24.8	1984

6. A newly discovered planet has a density that is 3/5 that of the Earth's density, $\rho_{E,}$ and a radius that is 3 times larger than the Earth's radius, R_E. What is the gravitational field strength at the surface of this newly discovered planet? Gravitational field strength of the new planet, $\boldsymbol{g_N}$, is $\boldsymbol{Fg/m}$ where the 2 interacting masses are the mass of the new planet, $\boldsymbol{M_N}$ and $\boldsymbol{m}$. At the surface, the distance between the 2 masses is $\boldsymbol{R_N}$, the radius of the new planet.

$$g_N = \frac{F_G}{m} = \frac{GM_N}{{R_N}^2}$$

We need to express $\boldsymbol{g}$ in terms of density, $\boldsymbol{\rho}$, rather than mass in this problem. $\rho = M/V$ and $M = \rho V$ where V is the volume of a sphere.

$$= \frac{G(\rho_N V_N)}{{R_N}^2} = \frac{G(\rho_N \frac{4}{3}\pi R_N^3)}{{R_N}^2} = \tfrac{4}{3}\pi G \rho_N R_N$$

$$= \tfrac{4}{3}\pi G(\tfrac{3}{5}\rho_E)(3R_E) = \tfrac{3}{5}(3)(\tfrac{4}{3}\pi G \rho_E R_E)$$

$$= \tfrac{9}{5} g_E = 17.64 N/kg$$

7. The Moon's orbit is roughly circular with an orbital radius of 3.84×10^{8} m. The Moon's mass is 7.35×10^{22} kg and the Earth's mass is 5.97×10^{24} kg. Calculate the Moon's orbital speed.

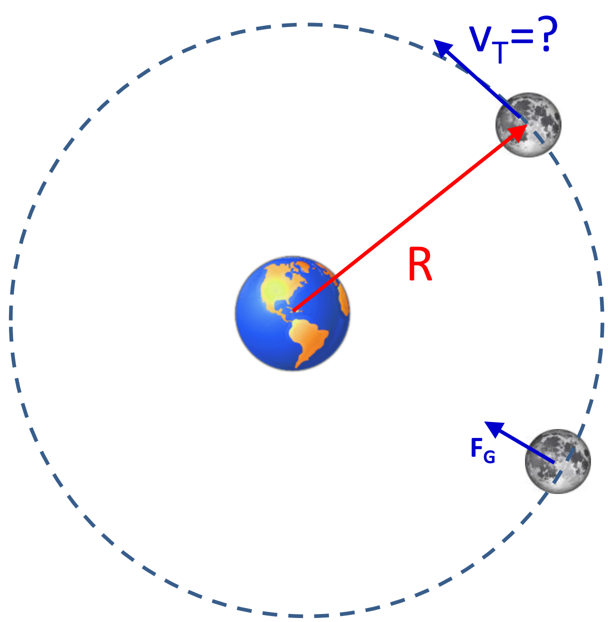
$$F_c = \sum F_r = ma_c = m_{moon}v^2/R$$

$$F_G = m_{moon}v^2/R$$

$$\frac{GM_E m_{moon}}{R^2} = \frac{m_{moon}v^2}{R}$$

$$\frac{GM_E}{R^2} = \frac{v^2}{R}$$

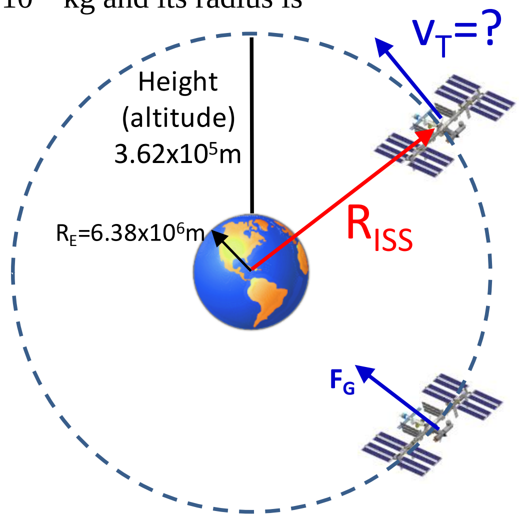
$$v = \sqrt{\frac{GM_E}{R}} = 1018 m/s$$



8.. The International Space Station orbits the Earth at an average altitude of 362 km. Assume that its orbit is circular, and calculate its orbital speed. The Earth's mass is 5.97×10^{24} kg and its radius is 6.38×10^{6} m

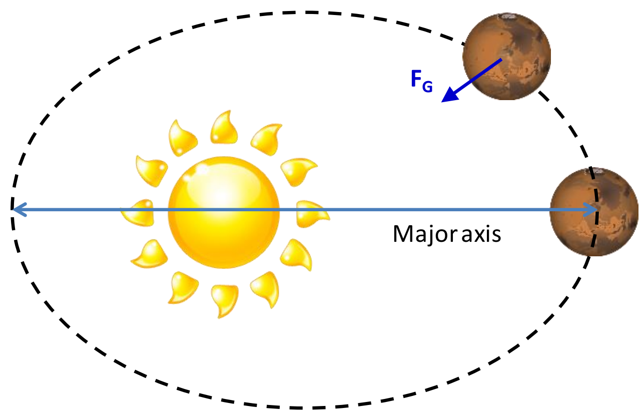
$R_{ISS} = R_E + h = (6.38+0.362) \times 10^6$ m $= 6.742 \times 10^6$ m

$$F_c = \sum F_r = ma_c = m_{ISS}v^2 / R_{ISS}$$
$$F_G = m_{ISS}v^2 / R_{ISS}$$
$$\frac{GM_E m_{ISS}}{R_{ISS}^{\ 2}} = \frac{m_{ISS}v^2}{R_{ISS}}$$
$$\frac{GM_E}{R_{ISS}^{\ 2}} = \frac{v^2}{R_{ISS}}$$
$$v = \sqrt{\frac{GM_E}{R_{ISS}}} = 7685 m/s$$



9. Mars orbits the Sun in about 5.94×10^{7} seconds (1.88 Earth years). (a) What is its semimajor axis (orbital radius) in meters? (The mass of the Sun is 1.99×10^{30} kg.) (b) What is Mars' semimajor axis (orbital radius) in AU? 1 AU = 1.50×10^{11} m. Distance between Earth and Sun is 1 AU = 1.50×10^{11} m.

$$F_c = \sum F_r = ma_c = m_{Mars}v^2 / R$$
$$F_G = m_{Mars}v^2 / R$$
$$\frac{GM_{sun}m_{Mars}}{R^2} = \frac{m_{Mars}v^2}{R} = \frac{m_{Mars}4\pi^2 R}{T^2}$$
$$\frac{GM_{sun}}{R^2} = \frac{4\pi^2 R}{T^2}$$
$$R^3 = \frac{GM_{sun}T^2}{4\pi^2}$$
$$R = 2.28x10^{11} m = 1.52 AU.$$

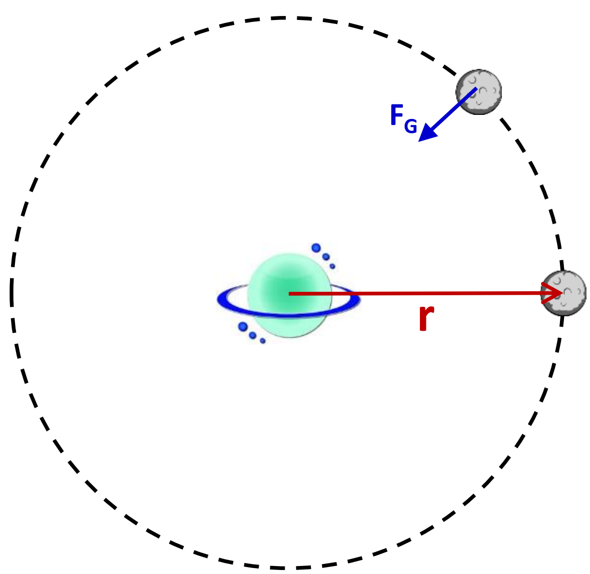


Semimajor axis = ½ Major axis ≈orbital radius, R

10. An extrasolar planet has a small moon, which orbits the planet in 336 hours. The orbital radius of the moon's orbit is $1.94x10^{9}$ m. What is the mass of the planet?

The moon's orbital period is
T = 336 hrs = 1.209×10^6 s

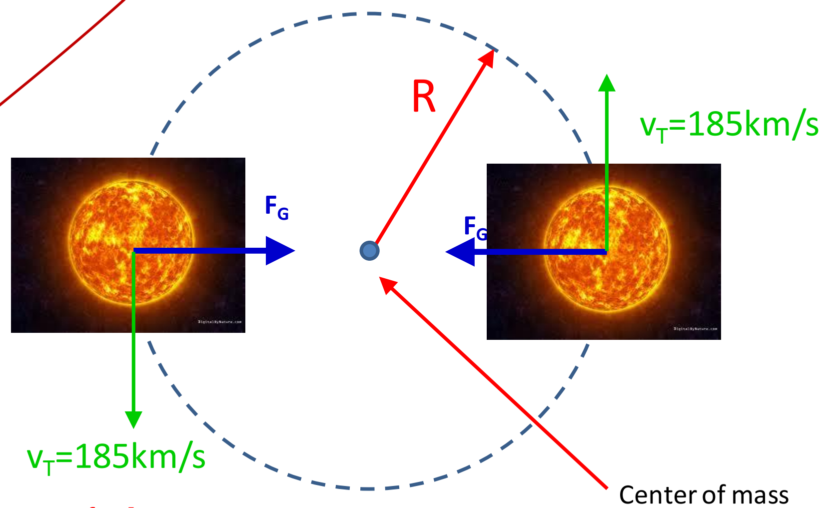
$$F_c = \sum F_r = ma_c = m_{moon}v^2 / R$$
$$F_G = m_{moon}v^2 / R$$
$$\frac{GM_{planet}m_{moon}}{R^2} = \frac{m_{moon}v^2}{R} = \frac{m_{moon}4\pi^2 R}{T^2}$$
$$\frac{GM_{planet}}{R^2} = \frac{4\pi^2 R}{T^2}$$
$$M_{planet} = \frac{4\pi^2 R^3}{GT^2} = 2.95x10^{27} kg$$



11. A certain binary star system consists of two identical stars in circular orbits about a common center of mass halfway between them. Their orbital speed is 185,000 m/s and each one orbits the center of mass in exactly 19 days. What is the mass of each star, in units of solar masses? The mass of the Sun is 1.99×10^{30} kg. (Hint: Draw a clear diagram. When setting the gravitational force equal to the centripetal force causing circular motion, you will see from your diagram that the radius of circular motion is NOT equal to the distance between the stars.)

Important note:
for Fc: the radius of circular motion for each star is R
for F_G: The distance between the stars is 2R and that is the distance over which Fg acts

$$F_c = \sum F_r = ma_c = m_{star}v^2 / R$$
$$F_G = m_{star}v^2 / R$$
$$\frac{Gm_{star}m_{star}}{(2R)^2} = \frac{m_{star}v^2}{R}$$
$$\frac{Gm_{star}}{4R^2} = \frac{v^2}{R}$$
$$m_{star} = \frac{4v^2R}{G}$$



Don't know R, but we know both v and T,so we can find R
T = 19 days = $1.6416x10^6$ s

$$v = \frac{2\pi R}{T}$$
$$R = \frac{vT}{2\pi} = 4.84x10^{10}m$$

$$m_{star} = \frac{4v^2R}{G}$$
$$= 9.93x10^{31}kg = 49.9M_{sun}$$

12. In June 2002, scientists at Caltech discovered a new orbiting body in the solar system, half the diameter of Pluto. Quaoar (KWAH-o-ar) takes 288 years to complete one orbit around the sun, and its orbit is remarkably circular. Find the distance from Quaoar to the sun (the sun's mass is 1.99×10^{30} kg). Assume a year has 365 days. Since Quaoar orbits the same central body as Earth, we could use Kepler's 3rd law to find the orbital radius of Quaoar if we were given the orbital radius of the Earth. However, all we are given is the mass of the sun (the central body), so we must use Newton's Laws of motion for circular motion and for gravitation.

$$F_c = \sum F_r = m_Q a_c = m_Q v^2 / R$$
$$F_G = m_Q v^2 / R$$
$$\frac{GM_{Sun}m_Q}{R^2} = \frac{m_Q v^2}{R} = \frac{m_Q 4\pi^2 R}{T^2}$$
$$\frac{GM_{Sun}}{R^2} = \frac{4\pi^2 R}{T^2}$$

orbital period of Quaoar is T=288 yrs = $9.082x10^9$ s

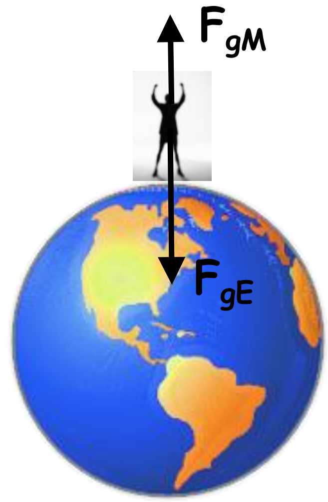
$$R^3 = \frac{GM_{Sun}T^2}{4\pi^2}$$
$$R = 6.50x10^{12}m$$

13. (<u>optional:</u> summing 2 gravitational forces acting on an object) The gravitational pull of the Moon is partially responsible for the tides of the sea. The Moon pulls on you, too, so if you are on a diet it is better to weigh yourself when this heavenly body is directly overhead! If you have a mass of 85.0 kg, how much less do you weigh if you factor in the force exerted by the Moon when it is directly overhead (compared to when it is just rising or setting)? Use the values 7.35×10^{22} kg for the mass of the moon, and 3.76×10^{8} m for its distance above the surface of the Earth. (For comparison, the difference in your weight would be about the weight of a small candy wrapper.)

True weight: direction is toward center of Earth

$$F_{GE} = mg = 85(9.8) = 833N$$

Fg due to moon overhead: direction is radially out



$$F_G = \frac{GM_{moon}m_{person}}{R^2} = \frac{(6.67x10^{-11})(7.35x10^{22})(85)}{(3.76x10^{8})^2}$$

$$= 0.00295N$$

That is how much less you would weigh

$$\boldsymbol{F_{G\,Total} = F_{G\,E} - F_{G\,M}}$$