Orbital Motion

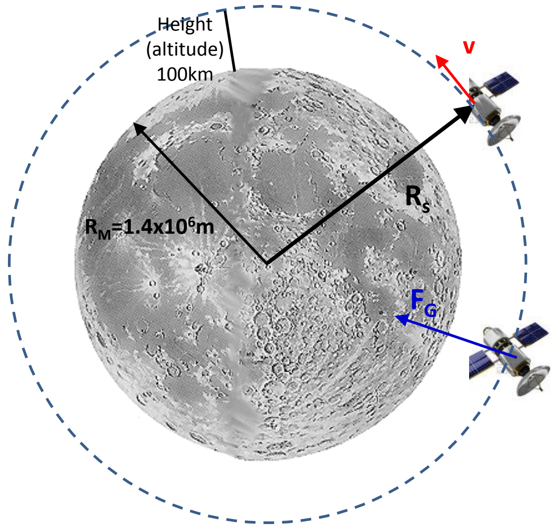
1. Suppose you are at mission control on the moon, in charge of launching a moon-orbiting communications satellite. The radius of the moon is $1.74x10^6$ m. The gravitational field strength on the surface of the moon is 1.6 m/s^2, or 1.6 N/kg.

a. Determine the mass of the moon.

$$g_M = \frac{GM_M}{R_M{}^2}$$

$$1.6 = \frac{(6.67x10^{-11})M_M}{(1.74x10^6)^2}$$

$$M_M = 7.26x10^{22}kg$$



b. The satellite is to have an altitude of 100 km above the moon's surface. What is the radius of the orbit of the satellite?

$$R_S = R_M + h = (1.74x10^6) + 100{,}000$$

$$R_S = 1.84x10^6 m$$

c. Determine the orbital velocity for the satellite. Don't know the period so cant use v=2πR/T.

$$F_c = \sum F_r = m_s a_c = \frac{m_s v^2}{R_S}$$

Satellite is in circular motion around the Moon so there is a centripetal force (gravity) acting on it

$$F_G = \frac{m_s v^2}{R_S}$$

$$\frac{GM_M m_s}{R_S{}^2} = \frac{m_s v^2}{R_S}$$

$$v = \sqrt{\frac{GM_M}{R_S}} = \sqrt{\frac{(6.67x10^{-11})(7.26x10^{22})}{(1.84x10^6)}} = 1622 m/s \quad (3626 mph)$$

d. How long will it take the satellite to orbit the moon? (This time is called the orbital period.)

$$v = \frac{2\pi R_S}{T}$$

$$T = \frac{2\pi R_S}{v} = \frac{2\pi(1.84x10^6)}{1622} = 7124s = 1.98hr$$

2. The space shuttle aims for an orbit about 500 km above the surface of the earth. The mass of the space shuttle is about 95,000 kg. The radius of the earth is approximately 6.38 x 10^6 m.

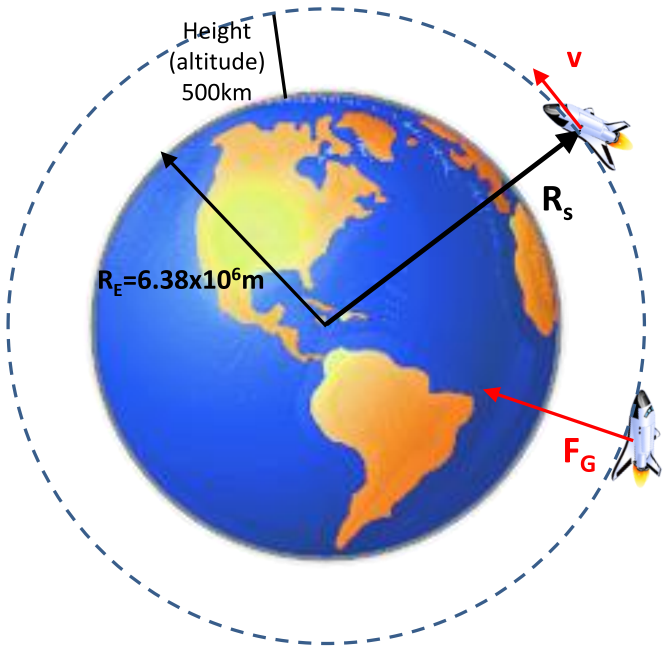
a. Determine the mass of the Earth.

$$g_E = \frac{GM_E}{R_E{}^2}$$

$$9.8 = \frac{(6.67x10^{-11})M_E}{(6.38x10^6)^2}$$

$$M_E = 5.98x10^{24} kg$$

b. Calculate the orbital speed of the space shuttle.
Shuttle is in circular motion around the Earth so there is a centripetal force (gravity) acting on it



$$F_c = \sum F_r = m_s a_c = \frac{m_s v^2}{R_S}$$

$$F_G = \frac{m_s v^2}{R_S}$$

$$\frac{GM_E m_s}{R_S{}^2} = \frac{m_s v^2}{R_S}$$

$$v = \sqrt{\frac{GM_E}{R_S}} = \sqrt{\frac{(6.67x10^{-11})(6x10^{24})}{(6.38x10^6 + 500{,}000)}}$$

$$v = 7627 m/s \quad (17{,}061 mph)$$

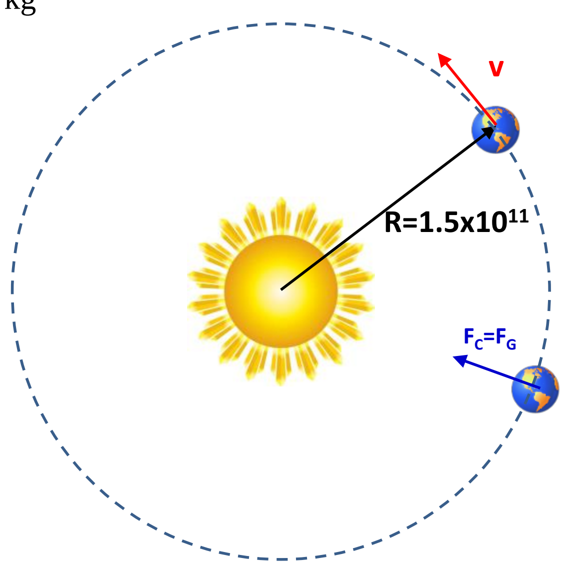
c. Calculate the orbital period of the space shuttle.

$$v = \frac{2\pi R_S}{T}$$

$$T = \frac{2\pi R_S}{v} = \frac{2\pi(6.88x10^6)}{7627} = 5665s = 1.57hr$$

3. The earth's orbit around the sun is very nearly circular, with an average radius of 1.5 x 10^{11} m. Assume the mass of the earth is 6 x 10^{24} kg and the mass of the sun is 1.99×10^{30}kg

a. Determine the average speed of the earth in its orbit around the sun.



$$F_c = \sum F_r = m_E a_c = \frac{m_E v^2}{R}$$

$$F_G = \frac{m_E v^2}{R}$$

$$\frac{GM_S m_E}{R^2} = \frac{m_E v^2}{R}$$

$$v = \sqrt{\frac{GM_S}{R}} = \sqrt{\frac{(6.67x10^{-11})(1.99x10^{30})}{(1.5x10^{11})}}$$

$$= 29{,}747 m/s \quad (66{,}542 mph)$$

b. What is the magnitude of the earth's average acceleration in its orbit around the sun?

$$a_C = \frac{v^2}{R} = \frac{29747^2}{1.5x10^{11}} = 0.0059m/s^2 \quad (0.06\%g)$$

c. Determine the gravitational force on the earth by the sun. How does the force on the earth by the sun compare to the force on the sun by the earth?

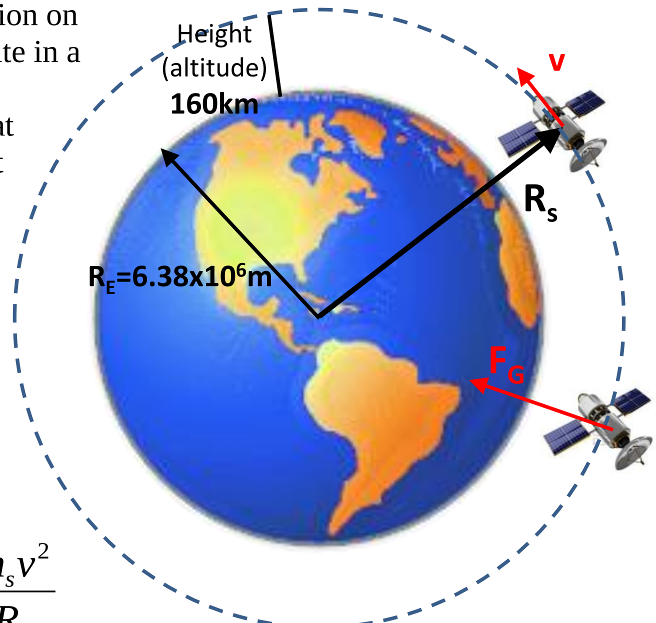
$$F_G = \frac{GM_S m_E}{R^2} = \frac{(6.67x10^{-11})(1.99x10^{30})(6x10^{24})}{(1.5x10^{11})^2} = 3.54x10^{22}N$$

Direction is towards the Sun.

Force on the Sun by the Earth is equal and opposite (Newton's 3rd Law)

$F_{G,\ Earth,\ Sun} = -F_{G,\ Sun,\ Earth} = 3.54 \times 10^{22} N$ towards the Earth

4. The National Academy of Science, in order to gather information on deforestation, wishes to place a 500 kg infrared-sensing satellite in a polar orbit around the earth. The radius of the earth is approximately 6.38 x 10^3 km, and the acceleration of gravity at the orbital altitude of 160 km is very nearly the same as it is at the surface of the earth.



a. Determine the orbital speed of the satellite as it orbits the earth.

Don't know the period so cant use v=2πR/T.
Shuttle is in circular motion around the Earth so there is a centripetal force (gravity) acting on it

First, you need the mass of the Earth since it was not given.

$$g_E = \frac{GM_E}{R_E^{\ 2}}$$

$$9.8 = \frac{(6.67x10^{-11})M_E}{(6.38x10^6)^2}$$

$$M_E = 5.98x10^{24}kg$$

$$F_c = \sum F_r = m_s a_c = \frac{m_s v^2}{R_S}$$

$$F_G = \frac{m_s v^2}{R_S}$$

$$\frac{GM_E m_s}{R_S^{\ 2}} = \frac{m_s v^2}{R_S}$$

$$v = \sqrt{\frac{GM_E}{R_S}} = \sqrt{\frac{(6.67x10^{-11})(5.98x10^{24})}{(6.38x10^6 + 160{,}000)}}$$

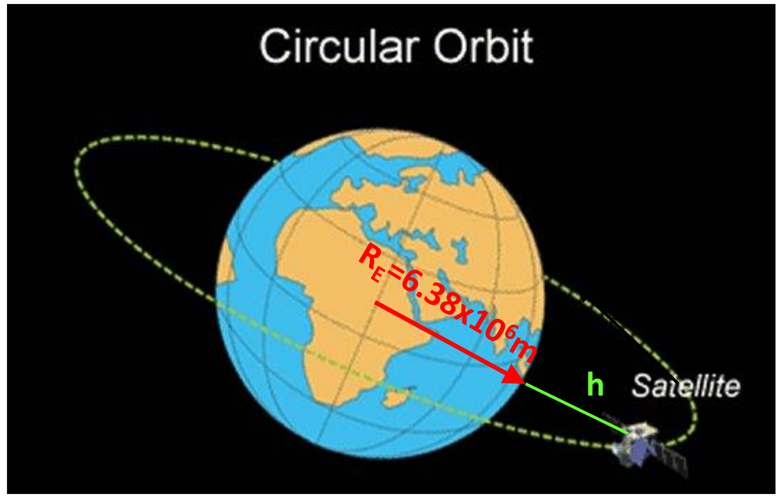
$$v = 7810m/s \quad (17{,}470mph)$$

b. Determine how long it would take for the satellite to make one complete revolution around the earth.

$$v = \frac{2\pi R_S}{T}$$

$$T = \frac{2\pi R_S}{v} = \frac{2\pi(6.54x10^6)}{7810} = 5259s = 1.46hr$$

5. A geosynchronous orbit is one in which a satellite in that orbit appears to be stationary relative to a point on the earth. In other words, it completes one orbit in the same time the earth takes to rotate about its axis, 24 hrs. Calculate the height of a satellite in a geosynchronous orbit. Use data for the earth's radius and mass from the previous problems.



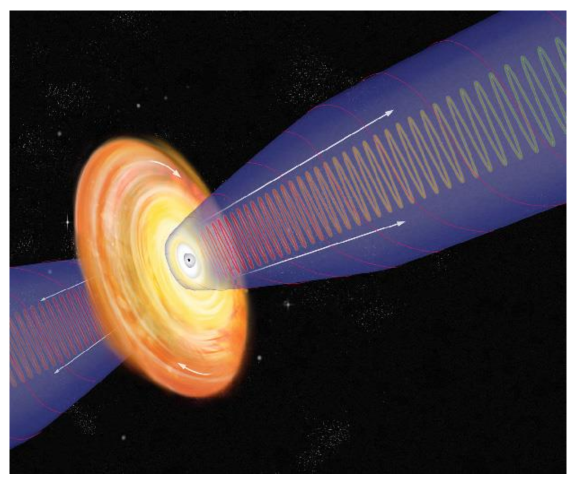
Orbital radius is R = R_E + h

For a geosynchronous satellite, T =24 hrs=86,400 s
$M_E = 5.98 \times 10^{24}$ kg
$R_E = 6.38 \times 10^6$ m

$$F_c = \sum F_r = m_s a_c = \frac{m_s v^2}{R}$$
$$F_G = \frac{m_s v^2}{R} = \frac{m_s 4\pi^2 R}{T^2}$$
$$\frac{GM_E m_s}{R^2} = \frac{m_s 4\pi^2 R}{T^2}$$
$$R^3 = \frac{GM_E T^2}{4\pi^2} = \frac{(6.67x10^{-11})(5.98x10^{24})(86400^2)}{4\pi^2}$$
$$R = 42.2x10^6 m$$

$$R = R_E + h$$
$$h = R - R_E = (42.2x10^6) - (6.38x10^6) = 35.8x10^6 m = 5.6R_E$$

6. Astronomers using the Hubble Space telescope have recently deduced the presence of an extremely massive core in the distant galaxy M87, so dense that it could well be a black hole (from which no light escapes). They did this by measuring the speed of gas clouds orbiting the core to be 780 km/s at a distance of 60 light years (5.7×10^{17} m) from the core. Deduce the mass of the core and compare it to the mass of our Sun (2×10^{30} kg).



An artist's concept shows the supermassive black hole in the heart of M87. The black hole is in the center of the orange disk. The disk itself is made of hot gas spiraling into the black hole. Powerful magnetic fields funnel some of the particles in the disk into jets (left and right of the disk) that extend hundreds of thousands of light-years into space. As the particles spiral through the magnetic fields, they emit radio waves.

$$F_c = \sum F_r = m_{gas} a_c = \frac{m_{gas} v^2}{R}$$
$$F_G = \frac{m_{gas} v^2}{R}$$
$$\frac{GM_{Core} m_{gas}}{R^2} = \frac{m_{gas} v^2}{R}$$
$$M_{core} = \frac{v^2 R}{G} = \frac{(780{,}000)^2 (5.7x10^{17})}{6.67x10^{-11}}$$
$$M_{core} = 5.20x10^{39} kg = 2.6x10^9 M_{Sun} = 2.6\ billion\ \ x M_{sun}$$

7. Find the centripetal acceleration for each of the following:

(a) a point on the earth's equator (ignore its orbital motion) (radius of Earth=6.38x10^6 m)

period of earth's rotation is 1 day = 86,400s

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 R_E}{T^2} = \frac{4\pi^2(6.38x10^6)}{86400^2} = 0.034 m/s^2 \approx 0.0034g$$

(b) the earth in its orbit around the sun (radius = 1.49x10^{11} m)

period of earth's rotation around sun is 1 year = 3.156 x 10^7 s

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(1.49x10^{11})}{(3.156x10^7)^2} = 0.0059 m/s^2 \approx 0.00059g$$

(c) the sun's orbit around the center of the Milky Way (period = 2 x10^{8}yrs; radius = 3 x10^{20} m)

period of earth's rotation around center of galaxy is 2x10^8 year = 6.31 x 10^{15} s

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(3x10^{20})}{(6.31x10^{15})^2} = 3.0x10^{-10} m/s^2 \approx 3.0x10^{-11}g$$

8. Here is the data for the space shuttle in its orbit around earth:

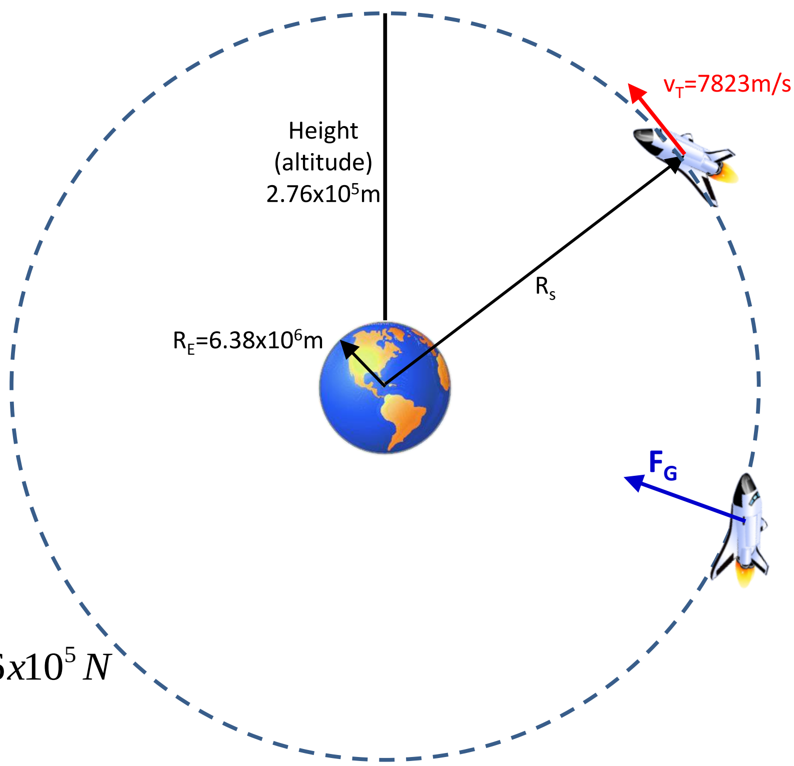
Shuttle mass in orbit, m_s = 94,802 kg
mass of the earth M_E = 6 x 10^{24} kg
shuttle orbital height above the earth = 2.76 x 10^{5}m
radius of the earth = 6.38 x10^6 m
shuttle tangential velocity when in orbit v_t = 7823 m/sec

a. Use the principles of circular motion to find the centripetal force necessary to keep the shuttle in its circular orbit around earth.

$$F_C = \sum F_r = ma_c = \frac{mv_t^2}{R_s{}^2}$$

$$= \frac{(94802)(7823^2)}{(6.38x10^6 + 0.276x10^6)}$$

$$= 8.72x10^5 N$$



b. Use Newton's Law of Universal Gravitation to find the gravitational force the earth exerts on the shuttle when the shuttle is in orbit.

$$F_G = \frac{GM_E m_s}{R_S{}^2}$$

$$= \frac{(6.67x10^{-11})(6x10^{24})(94802)}{(6.656x10^6)^2} = 8.56x10^5 N$$

Direction is towards the center of the Earth

c. How do the values calculated in parts *a* and *b* compare to one another? Explain.

$F_c > F_G$ by about 2%. The shuttle speed is a little more than that to be in stable circular orbit so shuttle is probable in an elliptical orbit. Moreover, there are other gravitational forces from other objects much farther away such as the moon, the sun and other planets and these would also affect the centripetal force on the shuttle in a minor way.