5. Answer the following based on the velocity vs. time graph.

a. Give a written description of the motion.
Object moves in negative direction at a constant speed of 4m/s for 2s
2-4sec, object continues to move in neg direction, but slows down to stop momentarily at 4s
4-8sec, object reverses direction to move in positive direction, speeding up to 8m/s by 8s.

b. Determine the displacement from t = 0s to t = 4 s.
\[ \Delta x_{0-4} = \text{Area under v-t from 0-4} = (-4)(2) + \frac{1}{2}(-4)(2) = -12 \text{ m} \]

c. Determine the displacement from t = 2 s to t = 6 s.
\[ \Delta x_{2-6} = \text{Area under v-t from 2-6} = \frac{1}{2}(-4)(2) + \frac{1}{2}(2)(4) = 0 \text{ m} \]

d. Determine the object’s acceleration at t = 4s.
\[ a_4 = \text{Slope of v-t at 4s} = 2 \text{ m/s}^2 \]

e. Sketch a possible x-t graph for the motion of the object. Explain why your graph is only one of many possible graphs.

6. Answer the following based on the position vs. time graph.

a. Where on the graph above is the object moving most slowly? (How do you know?)
B and F because slope, which represents velocity, is approximately 0

b. Between which points is the object speeding up? (How do you know?)
   
   between B and C,
   between C and D
   between F and G
   
   For all these intervals, the magnitude of the slope (velocity) is increasing (getting steeper)

c. Between which points is the object slowing down? (How do you know?)
   
   between A and B (slope is getting shallower)
   between D and E
   between E and F

   d. Where on the graph above is the object changing direction? (How do you know?)
      
      At B changes from positive to negative direction
      At F, changes from negative to positive direction

3. A car moves at 12 m/s and coasts up a hill with a uniform acceleration of -1.6 m/s²

   a) What is the displacement after 6 sec?
      
      \( v_0 = 12 \text{ m/s} \)
      \( v = \) \[ \Delta x = v_0 t + \frac{1}{2} a t^2 \]
      \( a = -1.6 \text{ m/s}^2 \)
      \( \Delta x = 12(6) + \frac{1}{2}(-1.6)(6)^2 \)
      \( t = 6 \text{ sec} \)
      \( \Delta x = 43.2 \text{ m} \)

   b) What is the displacement after 9 sec?
      
      \( v_0 = 12 \text{ m/s} \)
      \( v = \) \[ \Delta x = v_0 t + \frac{1}{2} a t^2 \]
      \( a = -1.6 \text{ m/s}^2 \)
      \( \Delta x = 12(9) + \frac{1}{2}(-1.6)(9)^2 \)
      \( t = 9 \text{ sec} \)
      \( \Delta x = 43.2 \text{ m} \)
c) What is going on? Plot v-t graph of this problem to explain. Determine Δx from the graph.

If you plot the v-t graph, you see that
at t=0, v = 12 and
the slope is the acceleration which is -1.6m/s².
Knowing the initial point and the slope, you can find the v at 6 sec and v at 9 sec.
The v-t graph shows that the car goes in the positive direction (up the hill), slowing down for the first 7.5sec. At 7.5 sec, it momentarily stops. After 7.5sec, it reverses direction and speeds up (rolls down the hill speeding up).
At 6 sec, car is on the way up and its displacement is 43.6 m (area under curve)
At 9 sec, car is rolling back down and is at the same position it was at 6 sec so its displacement is the same. (area under curve from 6-9 sec is 0).

4. A speedboat starts from rest and accelerates at +2.01 m/s² for 7.00 s. At the end of this time, the boat continues for an additional 6.00 s with an acceleration of +0.518 m/s². Following this, the boat accelerates at −1.49 m/s² for 8.00 s.
(a) What is the velocity of the boat at t = 21.0 s?
(b) Find the total displacement of the boat.
This is a 3 segment problem! The end velocity of the first segment is the start velocity of the second, the end velocity of the second is the start velocity of the third, etc

<table>
<thead>
<tr>
<th>7 sec</th>
<th>6 sec</th>
<th>8 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=2.01 m/s²</td>
<td>a=0.518 m/s²</td>
<td>a=-1.49 m/s²</td>
</tr>
<tr>
<td>v₀ = 0m/s</td>
<td>v₀ =</td>
<td>v₀ =</td>
</tr>
<tr>
<td>v =</td>
<td>v =</td>
<td>v =</td>
</tr>
<tr>
<td>a = 2.01m/s²</td>
<td>a = 0.518m/s²</td>
<td>a = -1.49m/s²</td>
</tr>
<tr>
<td>Δx =</td>
<td>Δx =</td>
<td>Δx =</td>
</tr>
<tr>
<td>t = 7.00s</td>
<td>t = 6.00s</td>
<td>t = 8.00s</td>
</tr>
</tbody>
</table>

There aren’t enough variables in the 2nd and 3rd acceleration parts to determine any unknowns. We must start in the 1st part and relate variables in the 1st part to those in the later parts. We know that the final velocity at the end of the 7 sec is the initial velocity at the start of the 2nd part. With that, we can find the final velocity of the 2nd part, which is also the initial velocity of the 3rd part.
\[ v = v_0 + at \]
\[ = 0 + (2.01)(7) \]
\[ = 14.07 m/s \]

\[ \Delta x = v_0 t + \frac{1}{2} at^2 \]
\[ = 0 + \frac{1}{2} (2.01)^2 \]
\[ = 49.2 m \]

\[ \Delta x_{\text{total}} = 49.2 + 93.7 + 89.8 = 232.7 m \]

5. **Challenging: Reaction Time Problem** A car is traveling 20 m/s when the driver sees a child standing on the road. She takes 0.8 s to react then steps on the brakes and slows at 7.0 m/s\(^2\). How far does the car go before it stops?

**Reaction time**

- \( a = 0 \)
- \( v = 20 m/s \)
- \( t = 0.8 s \)
- \( \Delta x = \) 

**Brake**

- \( a = -7 m/s^2 \)
- \( v = 0 \)
- \( \Delta x = \)

\[ \Delta x = vt = 16 m \]

\[ v^2 = v_0^2 + 2a\Delta x \]
\[ 0 = 20^2 + 2 (-7) \Delta x \]
\[ \Delta x = 28.6 m \]

\[ \Delta x_{\text{total}} = 16 + 28.6 = 44.6 m \]
6. On a planet that has no atmosphere, a rocket 14.2 m tall is resting on its launch pad. Freefall acceleration on the planet is 4.45 m/s\(^2\). A ball is dropped from the top of the rocket with zero initial velocity.

\[
\begin{align*}
&v_0 = 0 \\
v &= \\
a = -4.45 m/s^2 \\
\Delta y &= -14.2 m \\
t &= \\
v_f &= v_i + gt \\
&= 0 + (-4.45)(2.53 s) \\
&= -11.23 m/s
\end{align*}
\]

a) how long does it take to reach the launch pad

\[
\Delta y = v_0t + \frac{1}{2} gt^2 \\
-14.2 = 0 + \frac{1}{2}(-4.45)t^2 \\
t = 2.53 s
\]

b) what is the speed of the ball just before it hits the ground? (the speed is 11.23 m/s, the velocity is -11.23 m/s)

c) what is the speed of the ball just before it hits the ground? (the speed is 11.23 m/s, the velocity is -11.23 m/s)

7. You are a bungee jumping fanatic and want to be the first bungee jumper on Jupiter. The length of your bungee cord is 45.0 m. Free fall acceleration on Jupiter is 23.1 m/s\(^2\). What is the ratio of your speed on Jupiter to your speed on Earth when you have dropped 45 m? Ignore the effects of air resistance and assume that you start at rest.

\[
\begin{align*}
&\text{Jupiter} \\
v_0 = 0 \\
v &= \\
a = -23.1 m/s^2 \\
\Delta y = -45 m \\
t &= \\
v_f^2 &= v_i^2 + 2g\Delta y \\
v_f &= \sqrt{2g(-45)} \\
\end{align*}
\]

\[
\begin{align*}
&\text{Earth} \\
v_0 = 0 \\
v &= \\
a = -9.8 m/s^2 \\
\Delta y = -45 m \\
t &= \\
v_f^2 &= v_i^2 + 2g\Delta y \\
v_f &= \sqrt{2g(-45)} \\
\end{align*}
\]

\[
\frac{v_{f,Jupiter}}{v_{f,Earth}} = \sqrt{\frac{2g_{Earth}(-45)}{2g_{Jupiter}(-45)}} = \sqrt{\frac{23.1}{9.8}} = 1.54
\]
8. A stone is thrown vertically upwards with a speed of 20.0 m/s.
   a) How fast is it moving when it reaches a height of 12.0 m?

   \[ v_0 = 20 \text{ m/s} \]
   \[ v = v_0 + gt \]
   \[ a = -9.8 \text{ m/s}^2 \]
   \[ \Delta y = 12 \text{ m} \]
   \[ v^2 = v_0^2 + 2g\Delta y \]
   \[ t = \frac{v - v_0}{g} = \frac{20^2 + 2(-9.8)(12)}{196} \]
   \[ v = \sqrt{164.8} = \pm 12.8 \text{ m/s} \]

   There are 2 solutions:
   + 12.8 m/s when the stone is going up
   - 12.8 m/s when it is coming back down

   b) How long is required to reach this height?
      Time to get to 12 m on the way up:
      \[ v = v_0 + gt \]
      +12.8 = 20 - 9.8t
      \[ t = 0.73 \text{s} \]

      Time to get to 12 m on the way down
      \[ v = v_0 + gt \]
      -12.8 = 20 - 9.8t

   c) Why are there 2 answers to b? The rock has \( t = 3.35 \text{s} \) a displacement of +12 m twice: 12 m above its starting position on the way up and 12 m above its starting position on the way down.

9. A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 75.0 m high as shown at right.

   \[ v_0 = 12 \text{ m/s} \]
   \[ v = \]
   \[ a = -9.8 \text{ m/s}^2 \]
   \[ \Delta y = -75 \text{ m} \]

   a) How much later does it reach the bottom of the cliff?
   \[ \Delta y = v_0t + \frac{1}{2} gt^2 \]
   \[ -75 = 12t + \frac{1}{2}(-9.8)t^2 \]
   \[ 4.9t^2 - 12t - 75 = 0 \]

   This is a quadratic equation. You can solve it by using the quadratic formula to find the roots. The solution gives 2 roots:
   \[ t = 5.32 \text{ s} \] and \[ t_{\text{Rock}} = -2.88 \text{ s} \]
Since there is no negative time, we take the physically meaningful solution of \( t = 5.32 \text{ s} \)

b) What is its speed just before hitting?

\[ v = v_0 + gt \]
\[ = 12 - 9.8(5.32) \]
\[ v = -40.1 \text{ m/s} \]

c) What total distance did it travel? The total distance is the distance to the peak + distance back from the peak to top of cliff (same) + distance to the bottom of the cliff (75m). We first need to find \( \Delta y \) to peak of stone’s flight. At the peak, \( v = 0 \).

\[ v_0 = 12 \text{ m/s} \]
\[ v = 0 \]
\[ a = -9.8 \text{ m/s}^2 \]
\[ \Delta y = ? \]
\[ \Delta y = 7.35 \text{ m} \quad (to \ peak) \]

Total distance traveled = \( 7.35 + 7.35 + 75 = 89.7 \text{ m} \)