Geometry **3.3 Proving Lines Parallel** Name: Key

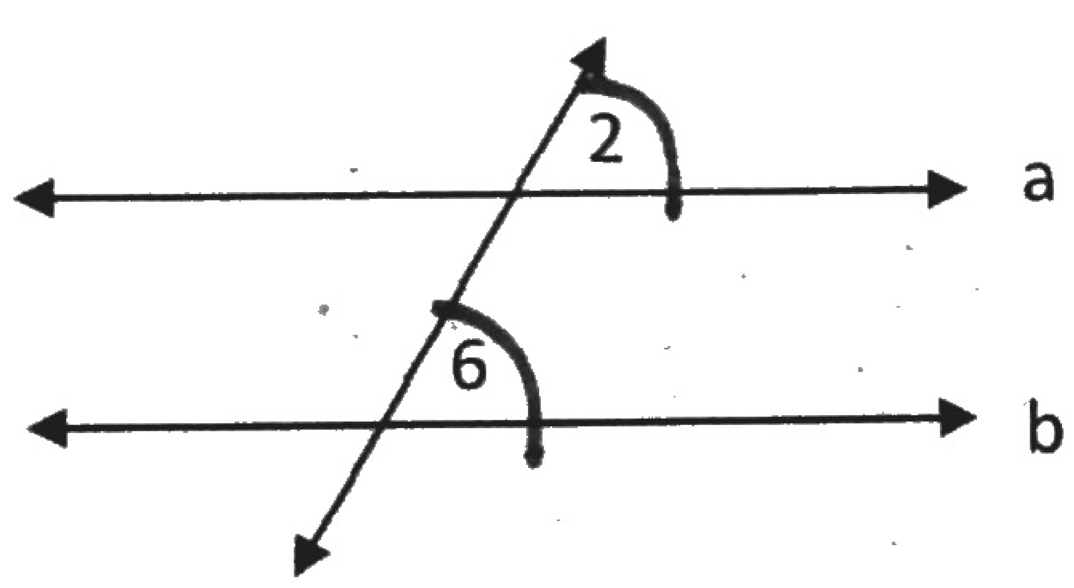
Recall: Parallel lines are lines that never intersect.

When parallel lines are intersected by a transversal...

1. Corresponding Angles are congruent.
2. Alternate Interior Angles are congruent.
3. Alternate Exterior Angles are congruent.
4. Same-Side Interior Angles are supplementary.

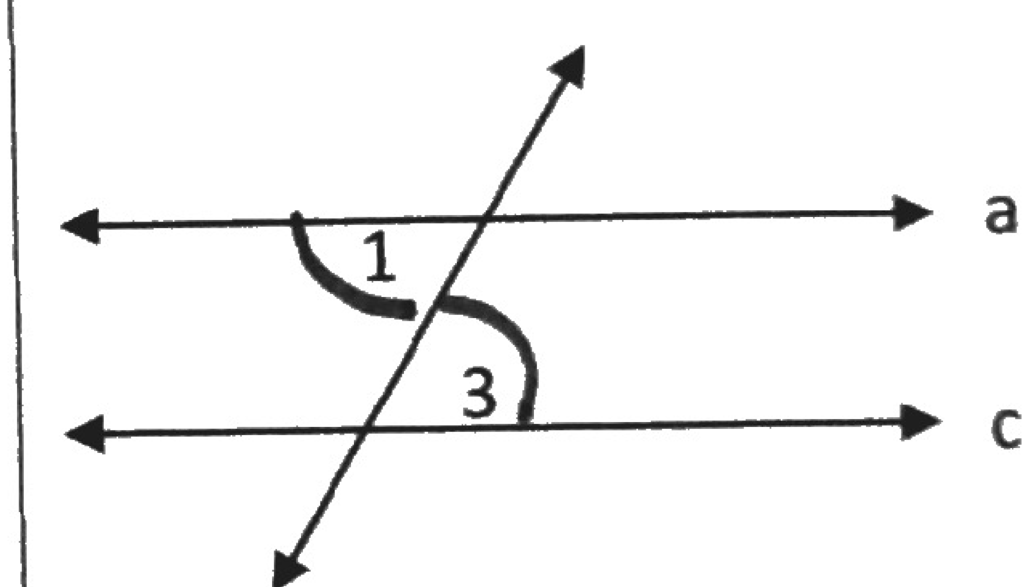
Today, we are going to focus on the converse of each of the above theorems. The CONVERSE of a theorem is found by switching the hypothesis and the conclusion. We will use today's information to help us prove that 2 lines are parallel.

Corresponding Angles Converse
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



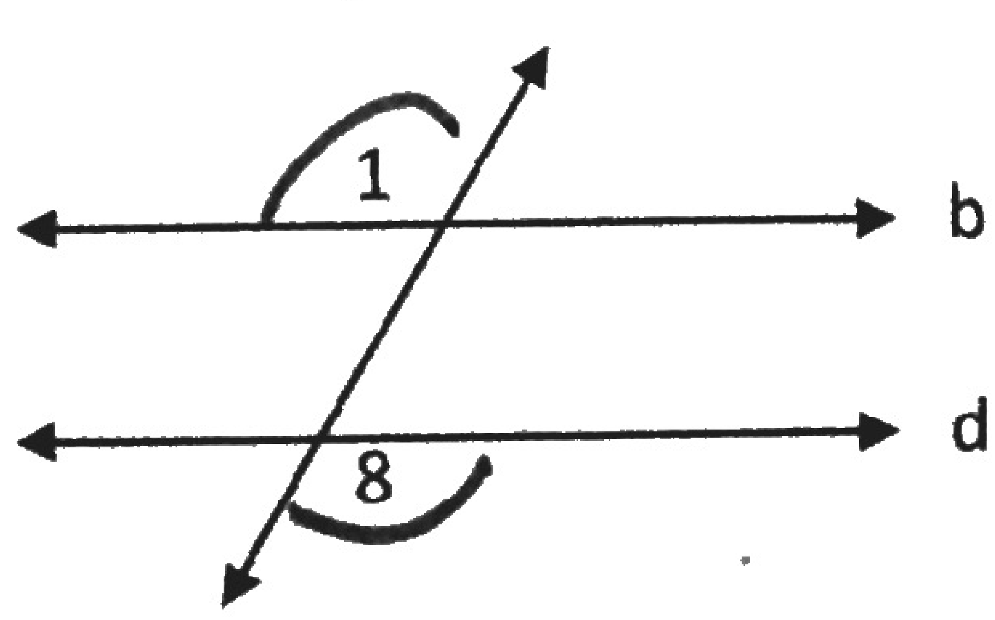
If $\angle 2 \cong \angle 6$, then $a \parallel b$.

Alternate Interior Angles Converse
If two lines are cut by a transversal so the alternate interior angles are congruent, then the two lines are parallel.



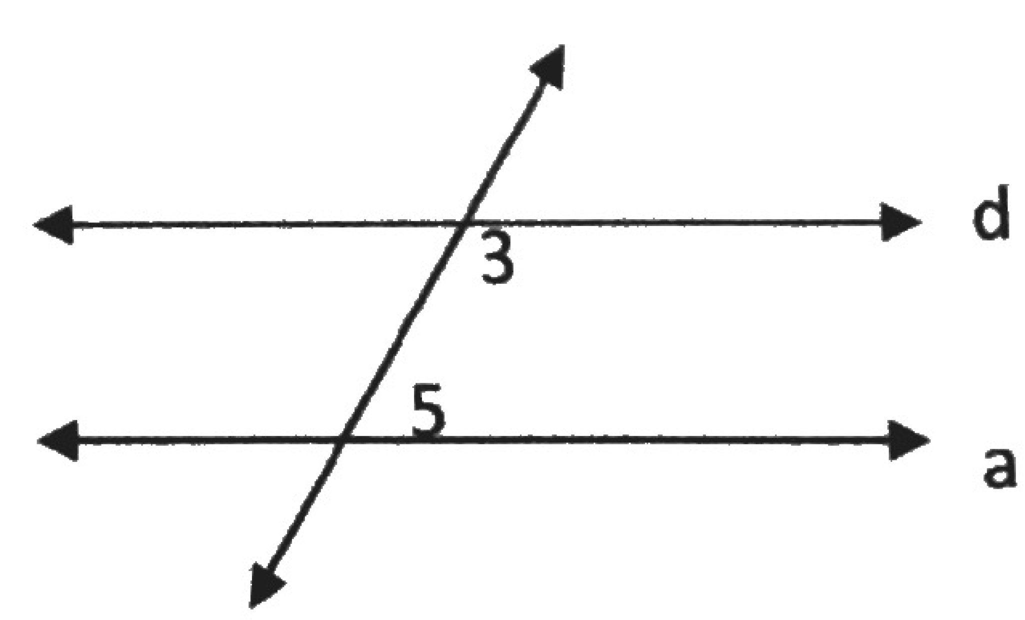
If $\angle 1 \cong \angle 3$, then $a \parallel c$.

Alternate Exterior Angles Converse
If two lines are cut by a transversal so the alternate exterior angles are congruent, then the two lines are parallel.



If $\angle 1 \cong \angle 8$, then $b \parallel d$.

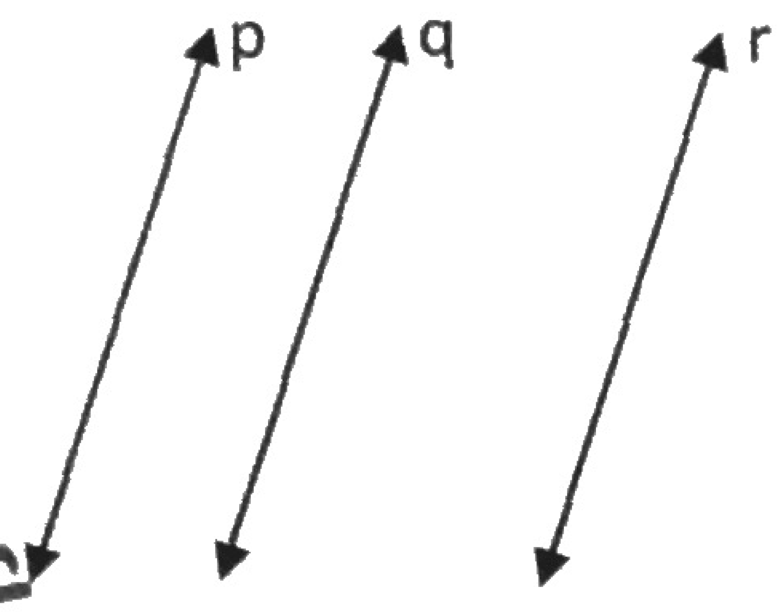
Same side
~~Consecutive~~ Interior Angles Converse
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the two lines are parallel.



If $m\angle 3 + m\angle 5 = 180$, then $a \parallel d$.

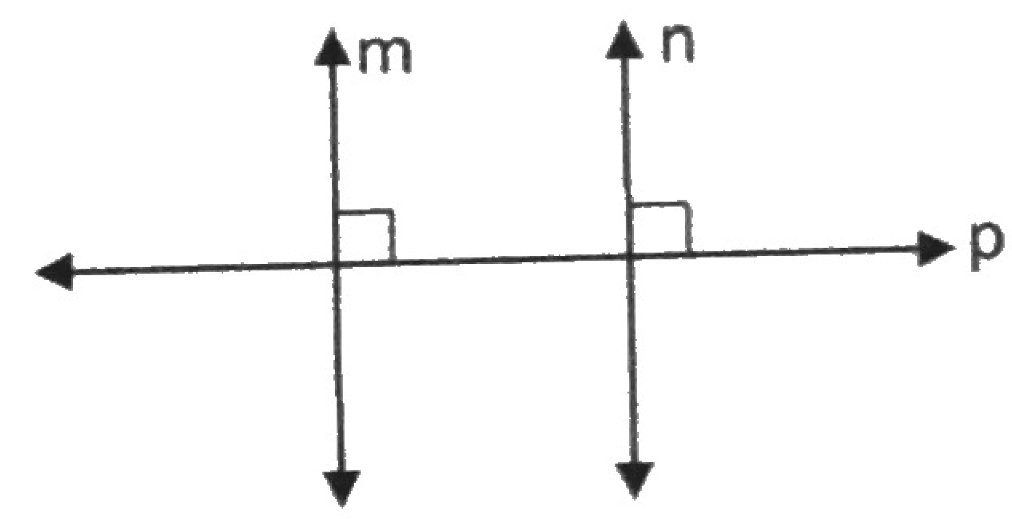
$a=b$, and $b=c \Rightarrow a=c$

Transitive Property of Parallel Lines
If two lines are parallel to the same line, then they are parallel to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$

Lines Perpendicular to a Transversal Theorem:
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

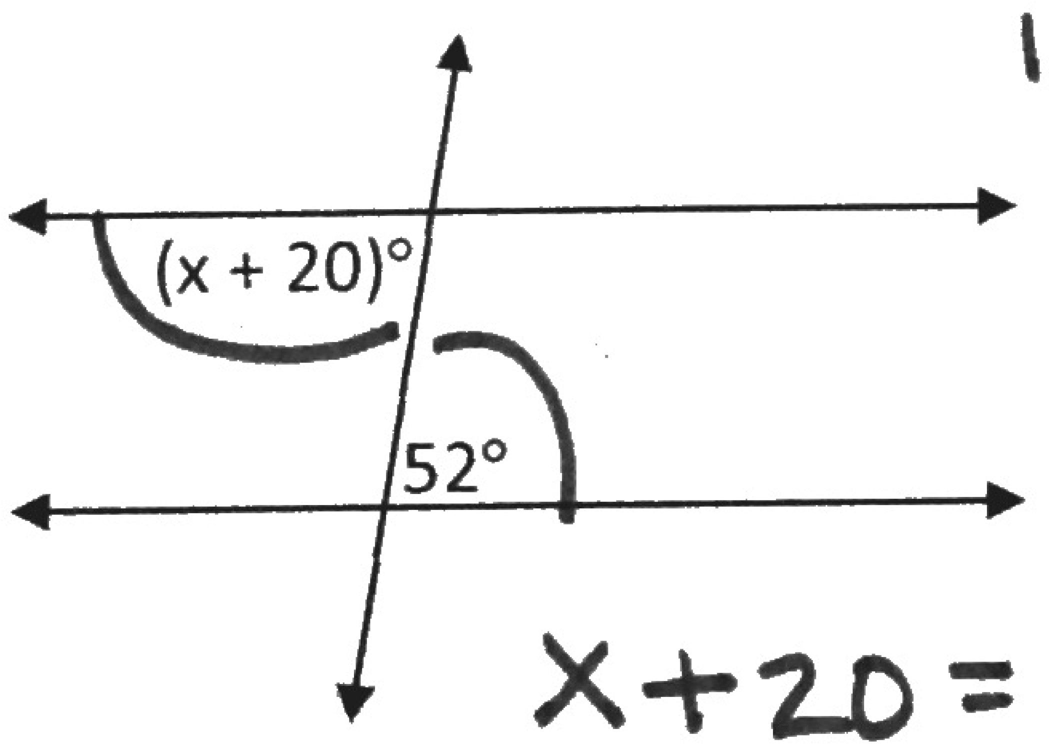


If m $\perp$ p and n $\perp$ p, then $m \parallel n$.

Practice Problems:

Find the value of x that makes the lines parallel.

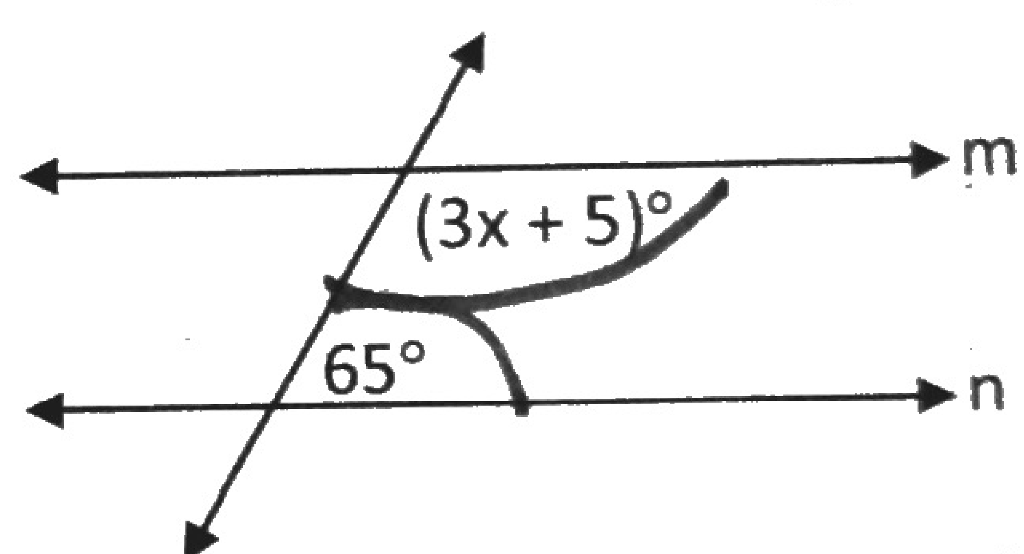
alternate interior need to be $\cong$.



$x+20=52$
$-20 \quad -20$
$\boxed{x=32}$

Find the value of x that makes m||n.

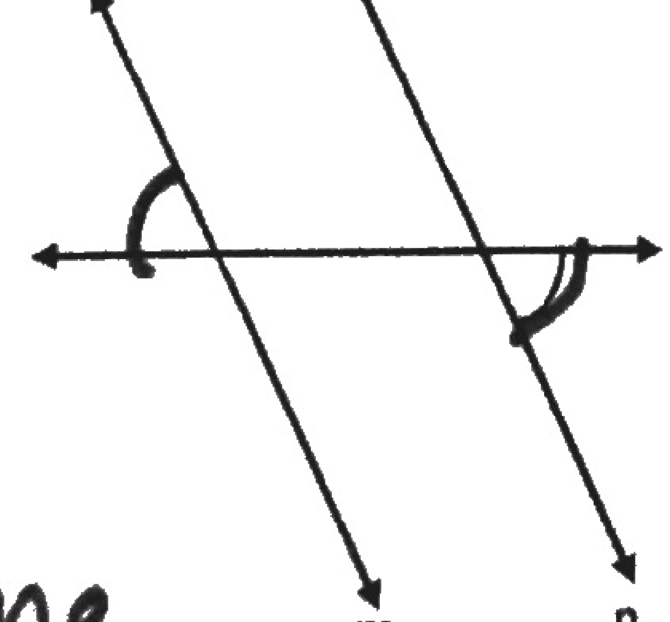
same side interiors need to be supplementary



$3x+5+65=180$
$3x+70=180$
$\frac{3x}{3}=\frac{110}{3}$
$\boxed{x=\frac{110}{3}}$

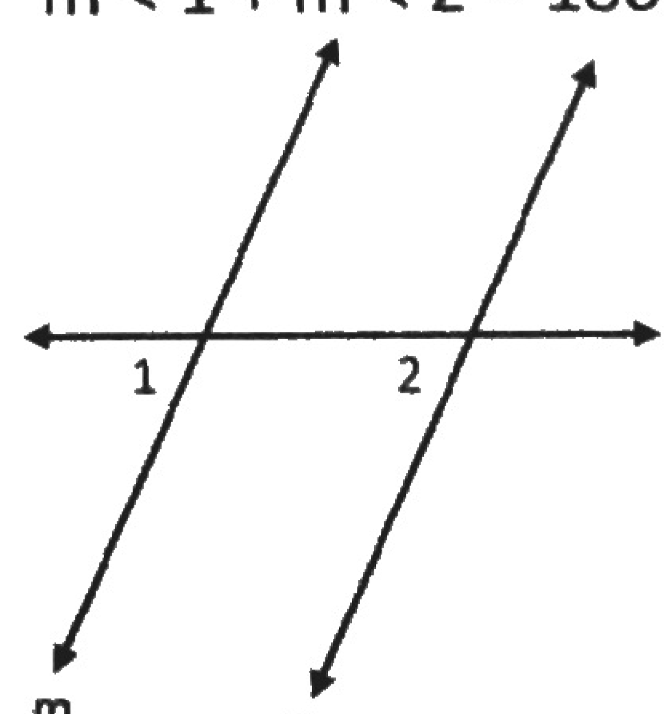
Can you prove that lines m and n are parallel? Explain why or why not.

a)



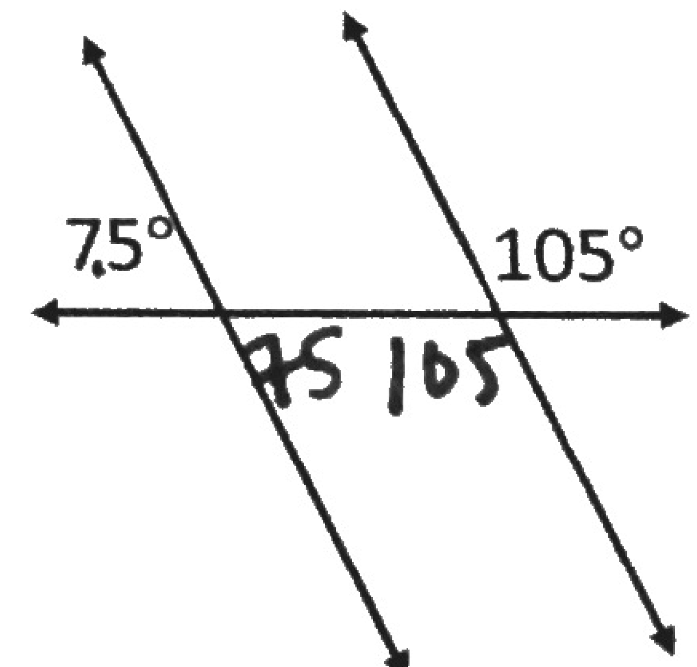
yes b/c the alternate exterior angles are congruent

b) m < 1 + m < 2 = 180°



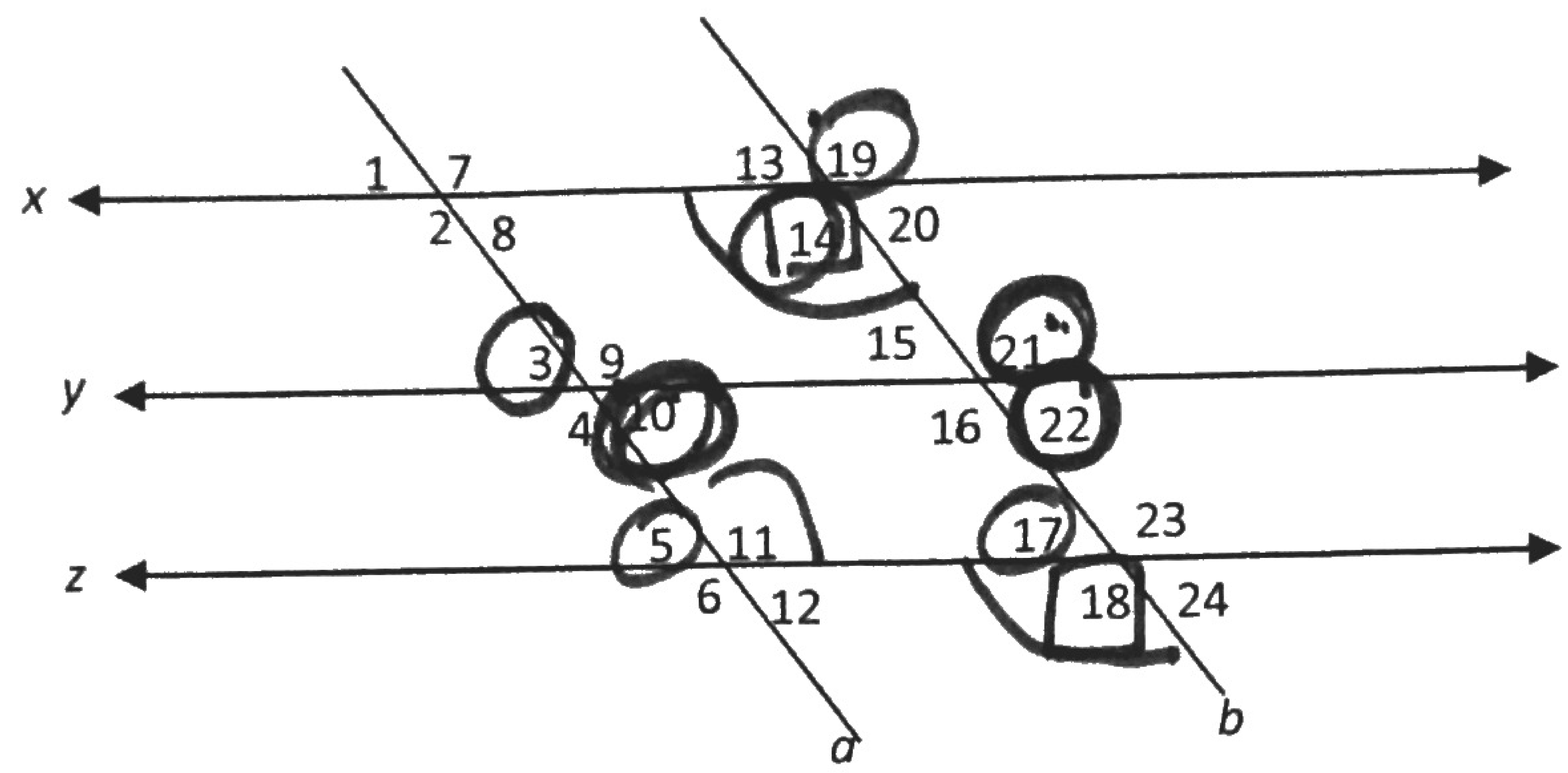
no, b/c I would need to know that $\angle 1 \cong \angle 2$

c)



Yes, b/c vertical angles $\Rightarrow$ same side interior angles are supplementary

Given the figure below, determine which lines are parallel (if any) based on the following:



1) $\angle 3 \cong \angle 22$

a ∥ b.

2) $\angle 14 \cong \angle 18$

x ∥ z

3) $m\angle 10 + m\angle 11 = 180°$

y ∥ z

4) $\angle 5 \cong \angle 17$

a ∥ b

5) $\angle 21 \cong \angle 10$

none

6) $\angle 14 \cong \angle 19$

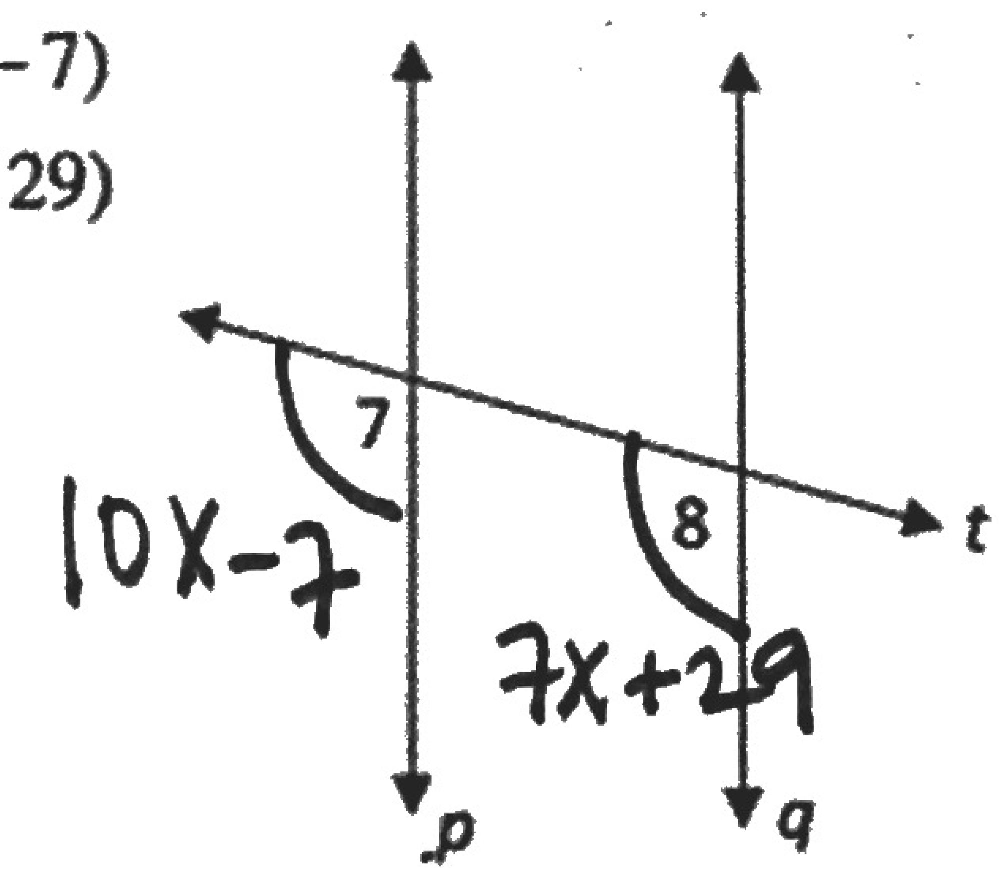
none.

7. Given: $m\angle 7 = (10x-7)$

$m\angle 8 = (7x+29)$

$x = 12$

Prove: $p \parallel q$



$10x - 7 = 7x + 29$

$10(12) - 7 = 7(12) + 29$

$120 - 7 = 84 + 29$

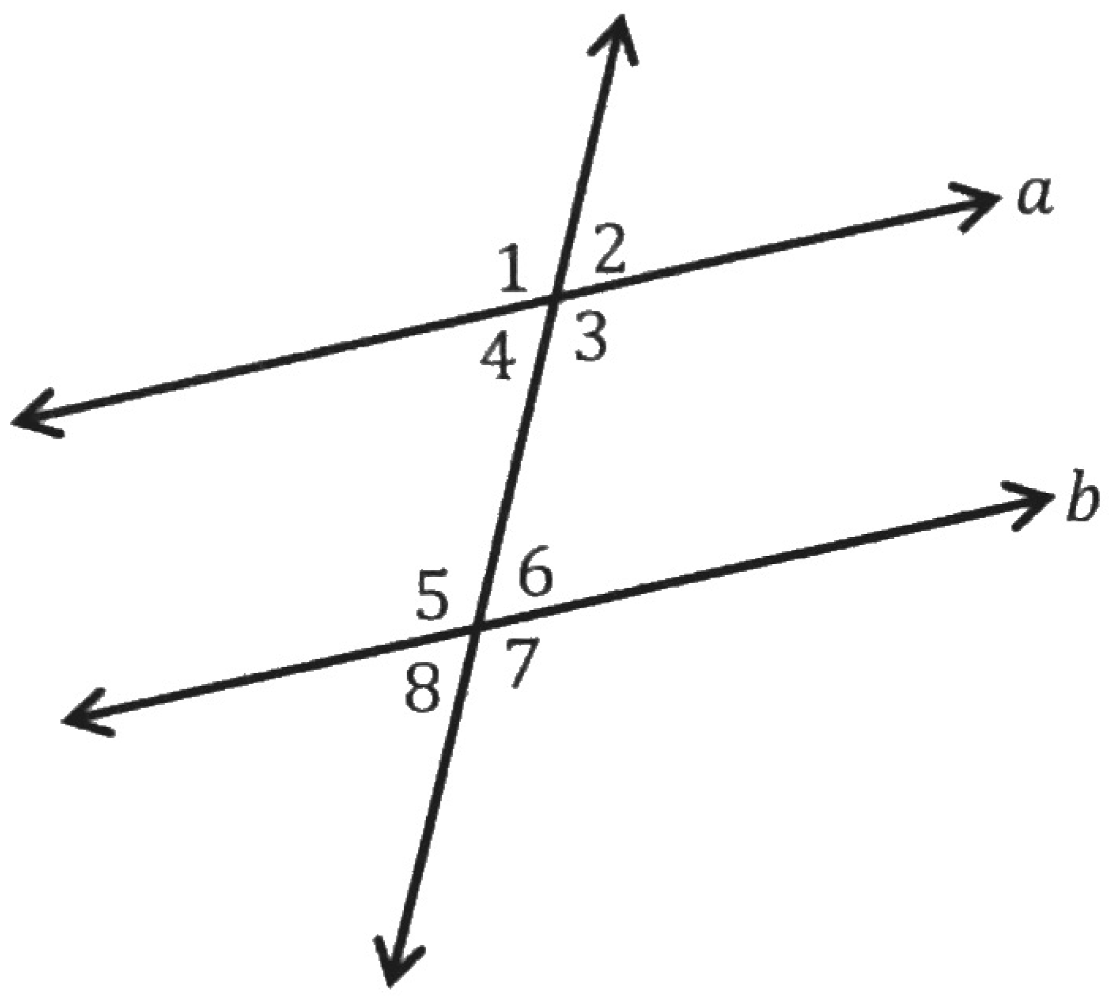
$113 = 113$ ✓

p ∥ q because corresponding angles are congruent.

8. Using the same picture, if m<7 = 110°, and m<8=115°, are lines p and q parallel? Why or why not?

No p ∦ q because corresponding angles are not congruent.

9. Find the value of x that would make $a \parallel b$.



a. $m\angle 2 = (8x - 1)°$ and $m\angle 6 = (23 - 4x)°$

$8x - 1 = 23 - \cancel{4x}$
$+4x \qquad +4x$

$12x - 1 = 23$
$+1 \qquad +1$

$\frac{12x}{12} = \frac{24}{12}$ $\boxed{x = 2}$

b. $m\angle 4 = [4(5x - 7)]°$ and $m\angle 6 = (10x + 2)°$

$4(5x - 7) = 10x + 2$

$20x - 28 = 10x + 2$
$-10x \qquad -10x$

$10x - 28 = 2$

$10x = 30$

$x = 3$

c. $m\angle 3 = (8x + 54)°$ and $m\angle 6 = (4x + 6)°$

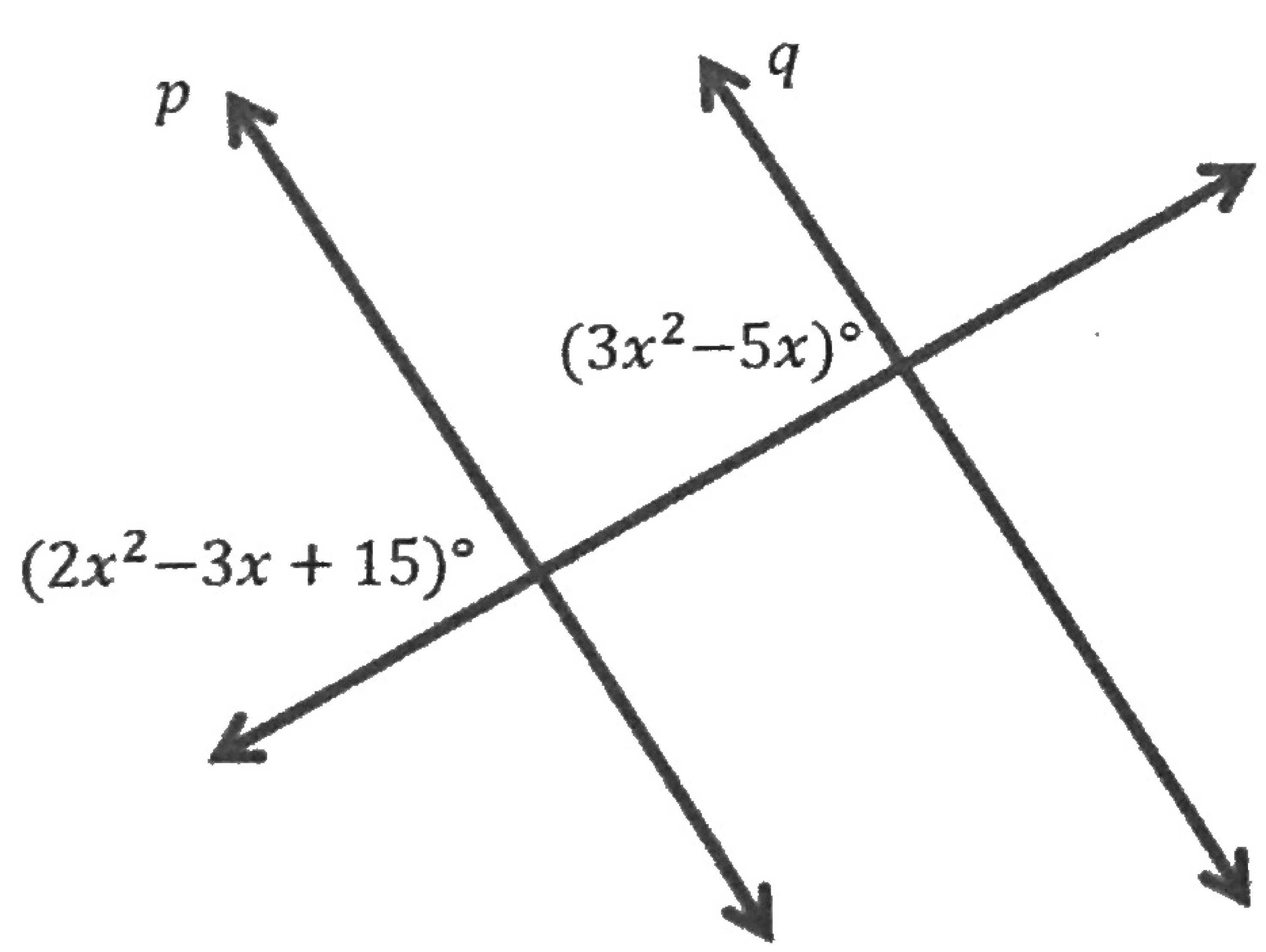
$8x + 54 + 4x + 6 = 180$

$12x + 60 = 180$

$12x = 120$

$x = 10$

10. Find the value(s) of x that make $p \parallel q$.



$3x^2 - 5x = 2x^2 - 3x + 15$
$-2x^2 \qquad -2x^2$

$x^2 - 5x = -3x + 15$
$+5x \qquad +5x$

$x^2 = 2x + 15$
$-2x - 15$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$x = 5$ and -3