Motion in Two Dimensions

The general equations of motion that we derived for motion in 1D:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2a\Delta x$$

Can be used to solve 2D problems by simply applying them separately in each of the two dimensions:

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2 \qquad \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x \qquad v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

Remember - these are not new equations! They are just equations you already know applied to a certain dimension.

Projectile Motion

Projectile - An object with motion that is being affected only by gravity.

Of course, in reality, projectiles are affected by air resistance.

However, when we do mathematical problems in class we will always assume that the effects of air resistance are small enough that we can ignore them.

Keep in mind that this is not necessarily true in the real world.

Projectile Motion

Projectile - An object with motion that is being affected only by gravity.

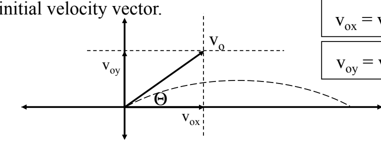
Because we are defining a projectile as something being affected only by gravity, we can learn more about its motion by thinking in terms of the horizontal and vertical components of the motion.

Gravity is the only force causing acceleration. Because gravity acts downward in the vertical, there is **vertical acceleration of 9.80 m/s² downward**.

Because there is no force acting horizontally, there is **no horizontal acceleration**.

Projectile Motion

v_{ox} and v_{oy} are simply the horizontal and vertical components of the initial velocity vector.



$v_{ox} = v_o\cos\theta$

$v_{oy} = v_o\sin\theta$

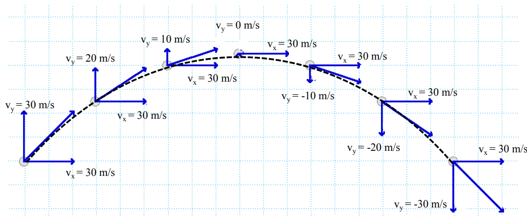
Also: remember that the acceleration in the horizontal direction is zero ($a_x = 0$).

$$\Delta x = v_{0x}t + \frac{1}{2}\cancel{a_x} t^2 \;(0) \qquad \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_x = v_{0x} + \cancel{a_x t} \;(0) \qquad v_y = v_{0y} + a_y t$$

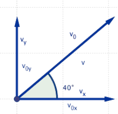
$$v_x^2 = v_{0x}^2 + \cancel{2a_x\Delta x} \;(0) \qquad v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

Projectile Motion



Because there is no horizontal acceleration, the horizontal component of the velocity stays constant. The vertical component of the velocity changes at a rate of 9.80 m/s² in the downward direction. As the ball is rising the vertical velocity decreases, at the top of the path it is zero, and on the way down it gets larger in the negative direction.

Projectile Motion



Notice how the velocity vector and its components change as the projectile moves. Does the horizontal component of the velocity change? Does the vertical component change? Does the total velocity vector change? When is the speed of the object smallest? When is the vertical component of velocity zero?

Solving Projectile Motion Problems

Read the problem and determine what information you are given. Look for given quantities like v_o, Θ , Δx, Δy, time, etc.

Make a sketch - set up axes with the origin at the launch site.

If you are given v_o, determine v_x and v_{oy} . If Θ is measured from the horizontal

$v_x = v_o \cos \Theta$ $\quad$ $v_{oy} = v_o \sin \Theta$

Keeping in mind the definition of a projectile, remember that $a_x = 0$ and $a_y = -9.80\ m/s^2$.

Solving Projectile Motion Problems

Use the general equations of motion applied in the horizontal (x) and vertical (y) directions.

$$\Delta x = v_{0x}t + \cancel{\frac{1}{2}a_x t^2}^{\,0} \qquad \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_x = v_{0x} + \cancel{a_x t}^{\,0} \qquad v_y = v_{0y} + a_y t$$

$$v_x^2 = v_{0x}^2 + \cancel{2a_x\Delta x}^{\,0} \qquad v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

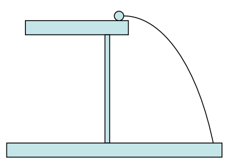
Use your given and the equations above to determine your unknowns. If you are asked for a total velocity or displacement, you will usually have to find the components first (using the equations above), and add the component vectors together to get the total.

Solving Projectile Motion Problems

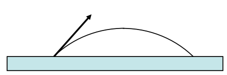
Common Problem Types:

Horizontal Launch:
In any problem where the object is thrown or launched horizontally:

$v_{oy} = v_o \sin(0^o)$
$v_{oy} = 0$



Angle Launch on Level Ground:
In any problem where the object returns to the same height from which it was thrown: $\Delta y = 0$

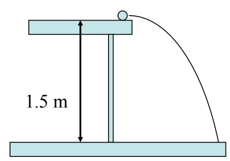


Projectile Problems

Example: Tom chases Jerry on a table 1.5 m high. Tom slides off at 5.0 m/s. Where does he land, and what are the velocity components as he hits?

Sketch:

1.5 m



Standard motion equations

$\Delta x = v_{ox}t + \cancel{(1/2)a_x t^2}^{\,0}$
$\Delta y = v_{oy}t + (1/2)a_y t^2$
$v_x = v_{ox} + \cancel{a_x t}^{\,0}$
$v_y = v_{oy} + a_y t$

Table top is horizontal, so initial velocity is all horizontal.

Given: $v_{oy} = 0$
$v_{ox} = 5\ m/s$
$\Delta y = -1.5\ m$
$a_x = 0$
$a_y = -9.80\ m/s^2$
$\Delta x = ?$
$\Delta x = v_x t$
$t = ?$

$\Delta y = \cancel{v_{oy}t}^{\,0} + (1/2)a_y t^2$
$\Delta y = (1/2)a_y t^2$

$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-1.5m)}{-9.80\ m/s^2}}$

$t = 0.5533\ s$

$x = v_x t = (5.0\ m/s)(0.5533s)$

$x = 2.77\ m$

$v_x = v_{ox} = 5\ m/s$

$v_y = v_{oy} + a_y t$

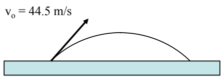
$v_y = (-9.8\ m/s^2)(0.5533\ s) = -5.42\ m/s$

Projectile Problems

Example: A baseball is hit with an initial speed of 44.5 m/s at an angle of 55^o from the horizontal. Find: a) horizontal range, and b) time in air.

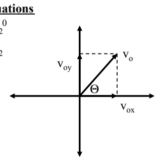
Sketch:

$v_o = 44.5\ m/s$



Standard motion equations

$\Delta x = v_{ox}t + \cancel{(1/2)a_x t^2}^{\,0}$
$\Delta y = v_{oy}t + (1/2)a_y t^2$
$v_x = v_{ox} + \cancel{a_x t}^{\,0}$
$v_y = v_{oy} + a_y t$



$v_{ox} = v_o\cos\Theta = 25.5\ m/s$
$v_{oy} = v_o\sin\Theta = 36.5\ m/s$
Level Ground, so $\Delta y = 0$
$\Delta x = v_{ox}t$
(don't know t!)

$\Delta y = v_{oy}t + (1/2)a_y t^2$
Level Ground, so $\Delta y = 0$

$-v_{oy}t = (1/2)a_y t^2$

$-v_{oy} = (1/2)a_y t$

$t = (-2v_{oy})/a_y = (-2)(36.5\ m/s)/(-9.80\ m/s^2)$

b) $t = 7.44\ s$

$x = v_{ox}t = (25.5\ m/s)(7.44\ s) = 190.\ m$